CONTEMPORARY MATHEMATICS

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Categorification and Higher Representation Theory

Anna Beliakova
Aaron D. Lauda
Editors

American Mathematical Society

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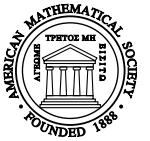

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Dedicated to Mikhail,
who taught us to look higher

Contents

Preface

The emergent mathematical philosophy of categorification is reshaping our view of modern mathematics by uncovering a hidden layer of structure in mathematics, revealing richer and more robust structures capable of describing more complex phenomena. This philosophy has led to a number of shocking new results and has ushered in a new area of interaction between algebra, geometry, and theoretical physics. Categorification is a powerful tool for relating various branches of mathematics and exploiting the commonalities between fields. It provides a language emphasizing essential features and allowing precise relationships between vastly different fields.

Categorified representation theory, or higher representation theory, aims to understand a new level of structure present in representation theory. Rather than studying actions of algebras on vector spaces where algebra elements act by linear endomorphisms of the vector space, higher representation theory describes the structure present when algebras act on categories, with algebra elements acting by functors. The new level of structure in higher representation theory arises by studying the natural transformations between functors. Often these natural transformations can be systematically described as representations of some monoidal category that categorifies the original algebra.

Representation theory has proven to be an especially fertile ground for categorification. This is in large part due to the geometric methods which pervade the subject. Geometric representation theory provides tools not only for classifying and studying representations but also for constructing natural bases with positivity and integrality properties. Through 'geometrization', various representation theoretic objects are realized via categories of sheaves on algebraic varieties. Passing from geometrization to categorification, these categories of sheaves are reinterpreted as categorical representations and the structure of functors between these categories, along with the higher structure of natural transformations, are abstracted and combinatorially encoded into higher categorical structures. Under categorification, basis elements are reinterpreted as indecomposable objects in a category, while structure constants become dimensions of Hom spaces. The positivity and integrality are then manifest. This enhanced perspective brings into play a powerful new set of tools that deepens our understanding of traditional representation theory.

This volume exhibits some of the current trends in higher representation theory and the diverse techniques that are being employed. These articles illustrate many important trends for the field including categorifications of quantum groups, Hecke algebras, and Heisenberg algebras, as well as related geometries. Some of the most pressing current directions are explored including roots of unity, the role of Cherednik algebras, higher analogs of crystal bases, interactions with modular

representation theory, geometric representation theory, and topology. This volume also showcases the many applications of higher representation theory to classical representation theory.

This volume was inspired by the conference Categorification in Algebra, Geometry, and Physics (a conference in honor of the 60th Birthday of Christian Blanchet), which took place at IESC, Menasina, Cargese, France, from May 4th to May 8th, 2015. This conference made clear that categorification is a rapidly emerging area of intense study. It also elucidated the need for a reference for newcommers to the field to learn the types of tools used in categorification, the problems where these tools have been successfully applied, and the future directions in which the field is moving. Our aim with this volume is to address this need. To this end, we have solicited articles from experts in categorification from around the world who were invited to share their unique perspective.

The organizers wish to thank the John Templeton Foundation and the University of Zurich for their generous support in making this conference possible.

Anna Beliakova
Aaron Lauda

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This volume exhibits some of the current trends in higher representation theory and the diverse techniques that are being employed in this field with the aim of showcasing the many applications of higher representation theory.

The companion volume (Contemporary Mathematics, Volume 684) is devoted to categorification in geometry, topology, and physics.

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