CONTEMPORARY MATHEMATICS

685

Algebraic and Geometric Methods in Discrete Mathematics

AMS Special Session on
Algebraic and Geometric Methods in
Applied Discrete Mathematics
January 11, 2015
San Antonio, TX

Heather A. Harrington
Mohamed Omar
Matthew Wright
Editors

American Mathematical Society

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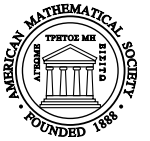

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Preface

This Contemporary Mathematics volume was initiated by the same named special session at the 2015 AMS/MAA Joint Mathematics Meetings, and also celebrates the themes of an AMS Mathematics Research Community (MRC) in 2014. The MRC on Algebraic and Geometric Methods in Applied Discrete Mathematics was held June 15–21, 2014, at Snowbird Resort in Utah. The goal of the MRC was to introduce young mathematicians to research bridging pure and applied mathematics—specifically, the use of algebraic and geometric methods to solve applied discrete problems. Working groups at the MRC included combinatorial topology in the social sciences, representation theory in data analysis, combinatorics in molecular biology, algebraic and geometric approaches in neuroscience, and algebraic and geometric methods in optimization.

The 2015 AMS/MAA Joint Mathematics Meetings, held January 10–13 in San Antonio, featured a special session dedicated to the themes of the MRC. Talks in this session were presented by experts in fields connected to the MRC topics. Given the intense interest in this special session and the enthusiasm of the session speakers and MRC participants, it was suggested that proceedings of the session be collected in a book. The present volume is that book.

We are deeply grateful to Carina Curto, Jesus A. De Loera, Christine Heitsch, Michael Orrison, and Francis Edward Su for organizing the MRC program and supporting this volume. We have benefited immensely from your mathematical expertise, mentoring, and willingness to introduce young mathematicians to your research areas. We also express great thanks to the AMS staff, especially Ellen Maycock, Melissa Colton, and Robin Hagan Aguiar for your work in arranging the logistics of the MRC, and to Christine Thivierge for your assistance in preparing this volume. The MRC would not have been possible without funding from the National Science Foundation, which we also acknowledge with thanks.

Heather A. Harrington
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CONM 685

Algebraic and Geometric Methods • Harrington et al., Editors

This volume contains the proceedings of the AMS Special Session on Algebraic and Geometric Methods in Applied Discrete Mathematics, held on January 11, 2015, in San Antonio, Texas.

The papers present connections between techniques from "pure" mathematics and various applications amenable to the analysis of discrete models, encompassing applications of combinatorics, topology, algebra, geometry, optimization, and representation theory. Papers not only present novel results, but also survey the current state of knowledge of important topics in applied discrete mathematics.

Particular highlights include: a new computational framework, based on geometric combinatorics, for structure prediction from RNA sequences; a new method for approximating the optimal solution of a sum of squares problem; a survey of recent Helly-type geometric theorems; applications of representation theory to voting theory and game theory; a study of fixed points of tensors; and exponential random graph models from the perspective of algebraic statistics with applications to networks.

This volume was written for those trained in areas such as algebra, topology, geometry, and combinatorics who are interested in tackling problems in fields such as biology, the social sciences, data analysis, and optimization. It may be useful not only for experts, but also for students who wish to gain an applied or interdisciplinary perspective.

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