CONTEMPORARY MATHEMATICS

700

Centre de Recherches Mathématiques Proceedings

CRM

Geometric and Computational Spectral Theory

Séminaire de Mathématiques Supérieures
Geometric and Computational Spectral Theory
June 15–26, 2015
Centre de Recherches Mathématiques,
Université de Montréal, Montréal, Québec

Alexandre Girouard
Dmitry Jakobson
Michael Levitin
Nilima Nigam
Iosif Polterovich
Frédéric Rochon
Editors

American Mathematical Society
Providence, Rhode Island

Centre de Recherches Mathématiques
Montréal, Québec, Canada

Geometric and Computational Spectral Theory

CONTEMPORARY MATHEMATICS

700

Centre de Recherches Mathématiques Proceedings

CRM

Geometric and Computational Spectral Theory

Séminaire de Mathématiques Supérieures
Geometric and Computational Spectral Theory
June 15–26, 2015
Centre de Recherches Mathématiques,
Université de Montréal, Montréal, Québec

Alexandre Girouard
Dmitry Jakobson
Michael Levitin
Nilima Nigam
Iosif Polterovich
Frédéric Rochon
Editors

American Mathematical Society
Providence, Rhode Island

Centre de Recherches Mathématiques
Montréal, Québec, Canada

Editorial Board of Contemporary Mathematics

EDITORIAL COMMITTEE

Dennis DeTurck, Managing Editor

Michael Loss　Kailash Misra　Catherine Yan

Editorial Committee of the CRM Proceedings and Lecture Notes

Jerry L. Bona
Vašek Chvatal
Galia Dafni
Nicole Tomczak-Jaegermann
Lisa Jeffrey
Ram Murty
Christophe Reutenauer
Pengfei Guan
Donald Dawson
Nicolai Reshetikhin
Héléne Esnault
Luc Vinet

2010 *Mathematics Subject Classification.* Primary 58Jxx, 35Pxx, 65Nxx.

Library of Congress Cataloging-in-Publication Data

Names: Girouard, Alexandre, 1976- editor.
Title: Geometric and computational spectral theory : Séminaire de Mathématiques Supérieures, June 15-26, 2015, Centre de Recherches Mathématiques, Université de Montréal, Montréal, Quebec, Canada / Alexandre Girouard, Dmitry Jakobson, Michael Levitin, Nilima Nigam, Iosif Polterovich, Frédéric Rochon, editors.
Description: Providence, Rhode Island : American Mathematical Society ; Montréal, Quebec, Canada : Centre de Recherches Mathématiques, [2017] | Series: Contemporary mathematics ; volume 700 | Includes bibliographical references.
Identifiers: LCCN 2017030037 | ISBN 9781470426651 (alk. paper)
Subjects: LCSH: Metric spaces–Congresses. | Spectral geometry–Congresses. | Geometry, Differential–Congresses. | AMS: Global analysis, analysis on manifolds – Partial differential equations on manifolds; differential operators – Partial differential equations on manifolds; differential operators. msc | Partial differential equations – Spectral theory and eigenvalue problems – Spectral theory and eigenvalue problems. msc | Numerical analysis – Partial differential equations, boundary value problems – Partial differential equations, boundary value problems. msc
Classification: LCC QA611.28 .G46 2015 | DDC 515/.353–dc23
LC record available at https://lccn.loc.gov/2017030037
DOI: http://dx.doi.org/10.1090/conm/700

Copying and reprinting. Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy select pages for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgment of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication is permitted only under license from the American Mathematical Society. Permissions to reuse portions of AMS publication content are handled by Copyright Clearance Center's RightsLink® service. For more information, please visit: http://www.ams.org/rightslink.

Send requests for translation rights and licensed reprints to reprint-permission@ams.org.

Excluded from these provisions is material for which the author holds copyright. In such cases, requests for permission to reuse or reprint material should be addressed directly to the author(s). Copyright ownership is indicated on the copyright page, or on the lower right-hand corner of the first page of each article within proceedings volumes.

© 2017 by the American Mathematical Society. All rights reserved.
∞ The paper used in this book is acid-free and falls within the guidelines established to ensure permanence and durability.

Visit the AMS home page at http://www.ams.org/

10 9 8 7 6 5 4 3 2 1　22 21 20 19 18 17

Contents

Preface

The 2015 edition of the *Séminaire de Mathématiques Supérieures* (SMS) took place on June 15-26, 2015, at the Université de Montréal. The topic of the meeting was *Geometric and computational spectral theory*, and it was perhaps the largest school in spectral geometry since the late nineties. The event was sponsored by the Centre de recherche mathématiques, as well as by the Fields Institute (Toronto), the Pacific Institute for the Mathematical Sciences (Vancouver), the Mathematical Sciences Research Institute (Berkeley), the Institut des sciences mathématiques (Montréal) and the Canadian Mathematical Society.

The summer school brought together students and internationally renowned experts in the geometric and computational aspects of spectral theory. The area of spectral theory has fascinated mathematicians and physicists for centuries, and recent years have seen remarkable progress in several branches of the field. The scientific program consisted of twelve minicourses in four main themes: geometry of eigenvalues, geometry of eigenfunctions, computational spectral theory, and spectral theory on singular spaces. A particular emphasis was made on the interplay between these topics, notably between the computational and the geometric part — which was one of the most novel aspects of the school. The minicourses were complemented by exercise sessions, computer labs and short presentations by selected junior participants who have already made important contributions to the subject. The school featured about 90 participants from 13 countries spanning five continents. The current volume contains the lecture notes and survey papers based on nine minicourses delivered at the summer school.

The lecture notes by Bruno Colbois (Neuchâtel) begin with an overview of some basic facts on the spectrum of the Laplace operator on Euclidean domains and Riemannian manifolds. The main part of the notes is concerned with the geometric eigenvalue inequalities, notably the Cheeger lower bound on the first eigenvalue and the Korevaar's method for estimating from above the eigenvalues of the Laplacian on a Riemannian manifold in a fixed conformal class. The last two sections cover some related recent results obtained by the author and his collaborators. In particular, bounds on the eigenvalues of hypersurfaces are discussed, as well as the notions of the topological and conformal spectra.

The lecture notes by Gregory Berkolaiko (Texas A & M) provide an introduction to spectral theory of Schrödinger operators on metric graphs, which in this setting are known as quantum graphs. A number of illuminating examples are treated in detail. A particular emphasis is made on the count of zeros of the eigenfunctions of quantum graphs.

A survey paper by Dorin Bucur (Chambéry) and Pedro Freitas (Lisbon) focusses on the celebrated Faber-Krahn inequality for the fundamental tone of the

Dirichlet Laplacian from the viewpoint of free boundary problems. The proof of the Faber-Krahn inequality which is presented is purely variational and, surprisingly, does not use rearrangement arguments. The advantage of this approach is its adaptability to extremal problems for higher eigenvalues and to more general Robin boundary conditions.

The paper by Pierre Bérard (Institut Fourier) and Bernard Helffer (Nantes and Orsay) is concerned with the nodal properties of the quantum harmonic oscillator and other Schrödinger operators on a Euclidean plane. In particular, the authors show that for the two-dimensional isotropic quantum harmonic oscillator there exists an infinite sequence of eigenfunctions with exactly two nodal domains. For the spherical Laplacian and the Dirichlet Laplacian on a square a similar result has been proved long time ago by A. Stern and H. Lewy. Other questions in nodal geometry are also discussed, such as bounds on the length of the zero set of a Schrödinger eigenfunction in the classically permitted region.

The next three contributions to the volume deal with various computational aspects of spectral theory. The paper by Jessica Bosch and Chen Greif (UBC) provides a review of the numerical methods for computing eigenvalues of matrices. A number of methods are discussed in detail, including the power method, the divide-and-conquer algorithms for tridiagonal matrices, as well as several approaches to compute the eigenvalues of large and sparce matrices, such as Lanczos, Arnoldi and Jacobi-Davidson methods.

The notes by Guido Kanschat (Heidelberg) give an introduction to the finite element methods for variational eigenvalue problems. In particular, Galerkin approximation is discussed in detail. Several eigenvalue problems are considered, including the standard boundary value problems for the Laplacian, as well as problems arising in the study of Maxwell and Stokes equations. In the appendix, some computer programs for experiments are presented.

The lecture notes by Alexander Strohmaier (Leeds) provide a link between the "numerical" and the "geometric" themes of this volume. The notes focus on the method of particular solutions and its applications to the computation of eigenvalues of the Laplace operator on Riemannian manifolds, including hyperbolic surfaces. Several Mathematica programs are included for illustration, which is most helpful. Computations of various spectral quantities, such as the spectral zeta function and the zeta-regularized determinant of the Laplace operator, are also discussed.

The remaining two contributions are based on minicourses requiring more advanced analytic background. The lecture notes by Daniel Grieser (Oldenburg) give an introduction to analysis on manifolds with corners in the spirit of R. Melrose. Its tools are used to construct quasimodes (i.e. approximate eigenfunctions) for degenerating families of domains, including adiabatic limit families and families exhibiting certain types of sclaing behaviour.

The notes by Colin Guillarmou (ENS) are concerned with geometric inverse problems on surfaces with boundary, such as the lens rigidity problem and its special case, the boundary rigidity problem. These problems naturally arise when one needs to recover the Riemannian metric on a surface from the boundary measurements. A closely related notion of the X-ray transform is also discussed.

The Proceedings of the *2015 Séminaire de Mathématiques Supérieures* on *Geometric and Computational Spectral Theory* cover a large variety of topics and methods, combining geometric, analytic and numerical ideas. We hope that this volume will serve both as a useful reference for experts and as an inspiring reading for young mathematicians who would like to learn more about this fascinating and rapidly developing area of mathematics. The editors would like to express their gratitude to all the contributing authors, as well as to all the speakers at the summer school. Last but not least, it is our pleasure to thank the SMS coordinator, Sakina Benhima, for all her hard work and help with the organization of this event.

Published Titles in This Subseries

700 **Alexandre Girouard, Dmitry Jakobson, Michael Levitin, Nilima Nigam, Iosif Polterovich, and Frédéric Rochon, Editors,** Geometric and Computational Spectral Theory, 2017

680 **Sergei Gukov, Mikhail Khovanov, and Johannes Walcher, Editors,** Physics and Mathematics of Link Homology, 2016

655 **A. C. Cojocaru, C. David, and F. Pappalardi, Editors,** SCHOLAR—a Scientific Celebration Highlighting Open Lines of Arithmetic Research, 2015

654 **Carlo Gasbarri, Steven Lu, Mike Roth, and Yuri Tschinkel, Editors,** Rational Points, Rational Curves, and Entire Holomorphic Curves on Projective Varieties, 2015

638 **Javad Mashreghi, Emmanuel Fricain, and William Ross, Editors,** Invariant Subspaces of the Shift Operator, 2015

630 **Pierre Albin, Dmitry Jakobson, and Frédéric Rochon, Editors,** Geometric and Spectral Analysis, 2014

622 **S. Ejaz Ahmed, Editor,** Perspectives on Big Data Analysis, 2014

606 **Chantal David, Matilde Lalín, and Michelle Manes, Editors,** Women in Numbers 2, 2013

605 **Omid Amini, Matthew Baker, and Xander Faber, Editors,** Tropical and Non-Archimedean Geometry, 2013

The Centre de Recherches Mathématiques (CRM) was created in 1968 to promote research in pure and applied mathematics and related disciplines. Among its activities are special theme years, summer schools, workshops, postdoctoral programs, and publishing. The CRM receives funding from the Natural Sciences and Engineering Research Council (Canada), the FRQNT (Québec), the NSF (USA), and its partner universities (Université de Montréal, McGill, UQAM, Concordia, Université Laval, Université de Sherbrooke and University of Ottawa).

The book is a collection of lecture notes and survey papers based on the mini-courses given by leading experts at the 2015 Séminaire de Mathématiques Supérieures on Geometric and Computational Spectral Theory, held from June 15–26, 2015, at the Centre de Recherches Mathématiques, Université de Montréal, Montréal, Quebec, Canada.

The volume covers a broad variety of topics in spectral theory, highlighting its connections to differential geometry, mathematical physics and numerical analysis, bringing together the theoretical and computational approaches to spectral theory, and emphasizing the interplay between the two.

American Mathematical Society
www.ams.org

Centre de Recherches Mathématiques
www.crm.math.ca

ISBN 978-1-4704-2665-1

9 781470 426651

CONM/700