

CONTEMPORARY MATHEMATICS

704

Advances in Ultrametric Analysis

14th International Conference

p-adic Functional Analysis

June 30–July 4, 2016

Université d'Auvergne, Aurillac, France

Alain Escassut

Cristina Perez-Garcia

Khodr Shamseddine

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Preface

This volume contains papers based on lectures given at the 14th International Conference on p -adic Functional Analysis which was held in Aurillac (France), at the Institut Universitaire de Technologie de Clermont-Ferrand (Université d'Auvergne) from June 30th to July 4th, 2016. The conference followed a tradition of thirteen earlier meetings which took place at Laredo (Spain, 1990), Santiago (Chile, 1992), Clermont-Ferrand (France, 1994), Nijmegen (The Netherlands, 1996), Poznan (Poland, 1998), Ioanina (Greece, 2000), Nijmegen (The Netherlands, 2002), Clermont-Ferrand (France, 2004), Concepción (Chile, 2006), East-Lansing (Michigan, USA, 2008), Clermont-Ferrand (France, 2010), Winnipeg (Manitoba, Canada, 2012), and Paderborn (Germany, 2014).

The 15 papers included in this book show recent developments in various areas of non-Archimedean analysis such as non-Archimedean Banach spaces, functions on Levi-Civita fields, spectral properties of ultrametric Banach rings and Banach algebras of differential functions, ultrametric Calkin algebras, new applications of the p -adic Nevanlinna theory, problems linked to dynamical systems on \mathbb{Q}_p , representation of p -adic functions with sub-coordinates, and techniques of summation for p -adic series. A survey paper recalls how ultrametric analysis appeared in the mathematical world and became an important domain of research; and it gives a nice classification of non-Archimedean fields.

The three editors are very grateful to the participants in the conference and to other colleagues who helped in the refereeing process, particularly José Aguayo, Jesus Araujo, Bertin Diarra, Helge Glöckner, Anatoly Kochubei, Albert Kubzdelá, Herminia Ochsenius, and Elena Olivos. We are also very grateful to Nicolas Mainetti, director of the Institut Universitaire de Technologie de Clermont-Ferrand and to the personnel of that institute (including students) for organizing the conference in Aurillac, a nice old city in Southern Auvergne.

Finally, we thank all the authors and referees as well as the American Mathematical Society, particularly Chistine Thivierge, for their role in producing this book of the proceedings of the conference, which is an important contribution to the knowledge of the latest results in ultrametric analysis.

The conference was dedicated to the memory of Lucien van Hamme, a professor at the Vrije Universiteit Brussel who worked in p -adic analysis until he retired in 2001. With his colleague Nicole de Grande de Kimpe he organized a conference on p -adic analysis at Hengelhof in 1986. From 1990 until 2002, he took part in all conferences of the new series begun at Laredo. He passed away on October 27, 2014. He was the thesis advisor of several students, particularly Ann Verdoodt and Stany Desmedt. With Nicole de Grande de Kimpe, he did great work in p -adic

analysis in Brussels. Our 14th Conference gave us the opportunity to honor his memory.

The next conference in the series is scheduled to be held in Poznan (Poland) in July, 2018.

Alain Escassut, Christina Perez-Garcia, Khodr Shamseddine

This book contains the proceedings of the 14th International Conference on p -adic Functional Analysis, held from June 30–July 4, 2016, at the Université d’Auvergne, Aurillac, France.

Articles included in this book feature recent developments in various areas of non-Archimedean analysis: summation of p -adic series, rational maps on the projective line over \mathbb{Q}_p , non-Archimedean Hahn-Banach theorems, ultrametric Calkin algebras, G -modules with a convex base, non-compact Trace class operators and Schatten-class operators in p -adic Hilbert spaces, algebras of strictly differentiable functions, inverse function theorem and mean value theorem in Levi-Civita fields, ultrametric spectra of commutative non-unital Banach rings, classes of non-Archimedean Köthe spaces, p -adic Nevanlinna theory and applications, and sub-coordinate representation of p -adic functions. Moreover, a paper on the history of p -adic analysis with a comparative summary of non-Archimedean fields is presented.

Through a combination of new research articles and a survey paper, this book provides the reader with an overview of current developments and techniques in non-Archimedean analysis as well as a broad knowledge of some of the sub-areas of this exciting and fast-developing research area.



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