CONTEMPORARY MATHEMATICS

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Surveys in Representation Theory of Algebras

Maurice Auslander Distinguished Lectures and International Conference Woods Hole Oceanographic Institute, Woods Hole, MA April 26–May 1, 2017 April 29–May 4, 2015 April 18–23, 2013

> Alex Martsinkovsky Kiyoshi Igusa Gordana Todorov Editors



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 $10 \ 9 \ 8 \ 7 \ 6 \ 5 \ 4 \ 3 \ 2 \ 1 \qquad 23 \ 22 \ 21 \ 20 \ 19 \ 18$

Contents

Preface	vii
On the abundance of silting modules LIDIA ANGELERI HÜGEL	1
Co-t-structures: The first decade PETER JØRGENSEN	25
Modules as exact functors MIKE PREST	37
A representation-theoretic approach to recollements of abelian categories CHRYSOSTOMOS PSAROUDAKIS	67
Going relative with Maurice—A survey ØYVIND SOLBERG	155
Coxeter groups and quiver representations HUGH THOMAS	173
Tree modules and limits of the approximation theory JAN TRLIFAJ	187

Preface

This volume is an outgrowth of some expository lectures from the annual Maurice Auslander Distinguished Lectures and International Conference. The latest event took place April 26 - May 1, 2017 at the Woods Hole Oceanographic Institute in Falmouth, MA. However, some of the presented material goes back to earlier years due to an extended editorial process which we hope has improved the contents of this volume.

The continued – and gratefully acknowledged – support from the National Science Foundation and Bernice Auslander results in a large number of graduate students and young researchers attending the conference. Their interest and unabating enthusiasm is *raison d'être* for this series of expository works (for the previous collections, see Contemporary Mathematics, vols. 607 and 673).

The selected expository lectures in this volume cover a wide area of research. Lidia Angeleri-Hügel gives a quick introduction to the recent concepts of silting and cosilting modules and their applications. Peter Jørgensen's brief survey of co-t-structures in triangulated categories comes with insightful philosophical observations. The reader who thinks they know what a module is may be surprised to learn about a more general point of view, as expounded in Mike Prest's paper. Chrysostomos Psaroudakis presents a comprehensive survey of recollements of abelian categories. The subject of relative homological algebra is brought back to life in Øyvind Solberg's survey. Hugh Thomas explains connections between Coxeter groups and representations of quivers. Jan Trlifaj's paper on tree modules explains recent striking results on approximation theory.

As we mentioned above, the primary audience for this volume are graduate students and young researchers working in representation theory. It is to be hoped that a much wider audience may also find this book useful.

The Editors

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This volume contains selected expository lectures delivered at the annual Maurice Auslander Distinguished Lectures and International Conference over the last several years.

Reflecting the diverse landscape of modern representation theory of algebras, the selected articles include: a quick introduction to silting modules; a survey on the first decade of co-t-structures in triangulated categories; a functorial approach to the notion of module; a representation-theoretic approach to recollements in abelian categories; new examples of applications of relative homological algebra; connections between Coxeter groups and quiver representations; and recent progress on limits of approximation theory.



