CONTEMPORARY MATHEMATICS

719

Unimodularity in Randomly Generated Graphs

AMS Special Session
Unimodularity in Randomly Generated Graphs
October 8–9, 2016
Denver, Colorado

Florian Sobieczky
Editor

AMS American Mathematical Society

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Dedicated to Prof. Russell Lyons in admiration of his work
and gratitude for his inspiration.

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Preface

'Well, this soup is inedible, again.'
'Why? This is even a very fine soup, today!'
'Nobody claims this isn't a fine soup.
Just that it is too hot to be edible.'
Karl Valentin, 'Der Hasenbraten'

The special session 'Unimodularity in randomly generated graphs' was held at the Denver AMS sectional meeting in October 2016. At this time of the year the Indian Summer turns the Aspen trees of the the Rocky Mountain Front Range into a golden curtain. It brought together people from different communities turning the discussion into a fruitful event.

Russ Lyons and Yuval Peres with the Rocky Mountains in the background

Unimodularity is a term originally used in the field of topological groups, where it refers to the equality of the left and right Haar measure. 'Unimodularity of graphs' was invented as a concept in percolation theory under the name of the 'Mass Transport Principle' where it was used with great success to generalise results from percolation on the Euclidean lattice to vertex-transitive graphs. It is equivalent to unimodularity of the automorphism group of the graph but has been developed further in terms of measures of rooted random graph valued processes. More generally, for measurable spaces of equivalence relations allowing graphings, unimodularity plays a central role in terms of the invariance and reversibility with respect to these equivalence relations. This is only a very incomplete glimpse of the range of topics in which unimodularity is the main underlying concept. Numerous related terms (such as hyperfiniteness, quasi-invariance, Diestel-Leader graphs and horocyclic products) are currently being developed further, and several of these developments are discussed in the articles of this volume. If different communities value different aspects of the same notion, let us remember the great Karl Valentin,

and try to understand the other specialist's view.

This special session and the publication of these proceedings have been supported by the Austrian Ministry for Transport, Innovation and Technology, the Federal Ministry of Science, Research and Economy, and the Province of Upper Austria in the frame of the COMET center SCCH.

Hagenberg, *Florian Sobieczky*, August 2018

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Unimodularity in Randomly Generated Graphs • Sobieczky, Editor

This volume contains the proceedings of the AMS Special Session on Unimodularity in Randomly Generated Graphs, held from October 8–9, 2016, in Denver, Colorado.

Unimodularity, a term initially used in locally compact topological groups, is one of the main examples in which the generalization from groups to graphs is successful. The "randomly generated graphs", which include percolation graphs, random Erdős–Rényi graphs, and graphings of equivalence relations, are much easier to describe if they result as random objects in the context of unimodularity, with respect to either a vertex-transient "host"-graph or a probability measure.

This volume tries to give an impression of the various fields in which the notion currently finds strong development and application: percolation theory, point processes, ergodic theory, and dynamical systems.

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