# **CONTEMPORARY MATHEMATICS**

## 722

## Arithmetic Geometry: Computation and Applications

16th International Conference Arithmetic, Geometry, Cryptography, and Coding Theory June 19–23, 2017 Centre International de Rencontres Mathématiques, Marseille, France

> Yves Aubry Everett W. Howe Christophe Ritzenthaler Editors



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#### Preface

The 16th edition of the AGC<sup>2</sup>T conference (Arithmetic, Geometry, Cryptography, and Coding Theory) took place at CIRM (Centre International de Rencontres Mathématiques) in Marseille, France, on June 19–23, 2017. This international conference has been a major event in the area of arithmetic geometry and its applications since 1987, and more than 94 participants joined us to celebrate its 30th anniversary. We thank all of them for creating a stimulating research environment during our week together.

The topics of the talks extended from algebraic number theory to diophantine geometry, and from curves and abelian varieties over finite fields to applications in codes and cryptography. We especially thank the speakers — Jeff Achter, Elise Barelli, Irene Bouw, Nils Bruin, Wouter Castryck, Alina Cojocaru, Mrinmoy Datta, Lucile Devin, Iwan Duursma, Elena Egorova, Sudhir Ghorpade, Marc Hindry, Nathan Kaplan, Valentijn Karemaker, Daniel Katz, Kiran Kedlaya, Pınar Kılıçer, Gilles Lachaud, Philippe Lebacque, Elisa Lorenzo García, Stefano Marseglia, Ivan Pogildiakov, Bjorn Poonen, Rachel Pries, Matthieu Rambaud, Alice Silverberg, Prasant Singh, Andrew Sutherland, Medhi Tibouchi, Andrey Trepalin, Michael Tsfasman, John Voight, Felipe Voloch, Marius Vuille, and Chia-Fu Yu — for their lectures.

As with any anniversary, it was a time for joy and exuberance, and some of the participants' memories of earlier editions of  $AGC^2T$  were recorded with the kind assistance of Stéphanie Vareilles and Guillaume Hennenfent.<sup>1</sup> It was also a moment for recollection and reflection, as the " $AGC^2T$  family" had lost one of its youngest and most brilliant members in the person of Alexey Zykin, who passed away with his wife Tatyana Makarova in a tragic accident a few months before the conference. With Alexey's friends and colleagues, we celebrated one more time his constant enthusiasm in all aspects of his research and life.

Gilles Lachaud, one of the founding fathers of the conference and one of its mainstays, passed away in February 2018. The next edition of the conference will have time set aside for memories of Gilles and for celebrations of his legacy. For now, we are honored to present one of his papers in this volume.

The editors would like to thank the staff of CIRM (Olivia Barbarroux, Muriel Milton, and Laure Stefanini) and of the Institut de Mathématiques de Marseille (Jessica Bouanane and Corinne Roux) for their remarkable professionalism and their friendly and constant help. We would also like to thank Christine Thivierge at the American Mathematical Society for guiding us through the *Contemporary Mathematics* production process. And of course, we give our deepest thanks to the

<sup>&</sup>lt;sup>1</sup>The video is available at https://youtu.be/KWklMv4Yya8.

#### PREFACE

authors of the papers in this volume for their mathematical creativity and for their patience with the editors.

Toulon, France San Diego, California, U.S.A. Rennes, France June 2018 Yves Aubry Everett Howe Christophe Ritzenthaler

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For thirty years, the biennial international conference AGC<sup>2</sup>T (Arithmetic, Geometry, Cryptography, and Coding Theory) has brought researchers to Marseille to build connections between arithmetic geometry and its applications, originally highlighting coding theory but more recently including cryptography and other areas as well.

This volume contains the proceedings of the 16th international conference, held from June 19–23, 2017.

The papers are original research articles covering a large range of topics, including weight enumerators for codes, function field analogs of the Brauer–Siegel theorem, the computation of cohomological invariants of curves, the trace distributions of algebraic groups, and applications of the computation of zeta functions of curves. Despite the varied topics, the papers share a common thread: the beautiful interplay between abstract theory and explicit results.



