

CONTEMPORARY MATHEMATICS

725

Nonlinear Dispersive Waves and Fluids

AMS Special Sessions on Spectral Calculus
and Quasilinear Partial Differential Equations,
and PDE Analysis on Fluid Flows
January 5–7, 2017
Atlanta, Georgia

Shijun Zheng
Marius Beceanu
Jerry Bona
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10 9 8 7 6 5 4 3 2 1 24 23 22 21 20 19

Contents

Preface	vii
Combined list of speakers	ix
Blowup rate for mass critical rotational nonlinear Schrödinger equations NYLA BASHARAT, YI HU, and SHIJUN ZHENG	1
Gradient estimates for weak solutions of linear elliptic systems with singular-degenerate coefficients DAT CAO, TADELE MENGESHA, and TUOC PHAN	13
Virial estimates for hard spheres RYAN DENLINGER	35
The almost global existence to classical solution for a 3-D wave equation of nematic liquid-crystals YI DU, GENG CHEN, and JIANLI LIU	53
Instability of solitons–revisited, I: The critical generalized KdV equation LUIZ GUSTAVO FARAH, JUSTIN HOLMER, and SVETLANA ROUDENKO	65
Instability of solitons–revisited, II: The supercritical Zakharov-Kuznetsov equation LUIZ GUSTAVO FARAH, JUSTIN HOLMER, and SVETLANA ROUDENKO	89
Stabilization of dispersion-generalized Benjamin-Ono CYNTHIA FLORES, SEUNGLY OH, and DEREK SMITH	111
Uniqueness of standing-waves for a non-linear Schrödinger equation with three pure-power combinations in dimension one DANIELE GARRISI and VLADIMIR GEORGIEV	137
Below-threshold solutions of a focusing energy-critical heat equation in \mathbb{R}^4 STEPHEN GUSTAFSON and DIMITRIOS ROXANAS	149
A regularity upgrade of pressure DONG LI and XIAOYI ZHANG	163
On large future-global-in-time solutions to energy-supercritical nonlinear wave equation SHUANG MIAO	187
The nonlinear Schrödinger equation with an inverse-square potential JASON MURPHY	215

The harmonic map heat flow on conic manifolds YUANZHEN SHAO and CHANGYOU WANG	227
On the global regularity issue of the two-dimensional magnetohydrodynamics system with magnetic diffusion weaker than a Laplacian KAZUO YAMAZAKI	251
Orbital stability of standing waves for fractional Hartree equation with unbounded potentials JIAN ZHANG, SHIJUN ZHENG, and SHIHUI ZHU	265

Preface

This volume stems from the two special sessions “Spectral Calculus and Quasi-linear Partial Differential Equations” and “PDE Analysis on Fluid Flows” that took place at the AMS-MAA Joint Mathematics Meetings held in Atlanta, Georgia, January 4-7, 2017, where the participants include many well-known mathematicians as well as promising junior researchers in the field. The contributions therein review some of the recent developments in the mathematical theory of quantum waves and fluids. In particular, they reveal a small fraction of the facet of the current research topics that have stayed in the heart of the flow at its golden age.

The special proceedings features invited contributions of research papers and survey articles reflecting the frontier studies in evolutionary differential equations. The topics of the expositions include spectral and nonlinear analysis of PDEs arising in quantum mechanics, plasma, fluid mechanics, water waves (oceanology), crystallization, stellar dynamics (astrophysics) and other applied science.

The paper of Y. Du, G. Chen and J. Liu studies the global existence for wave equation of liquid crystals. S. Miao considers the existence of large data solution for NLW equation in the energy-supercritical regime. The paper of J. Murphy gives a survey on the scattering versus blowup results for NLS with an inverse square potential in the energy-subcritical case, while the paper of S. Gustafson and D. Roxanas surveys on global existence and blowup for nonlinear heat equation in the energy-critical case.

D. Garrisi and V. Georgiev prove the uniqueness of standing waves for NLS with combined power nonlinearities. J. Zhang, S. Zhu and S. Zheng show the orbital stability for standing waves for the Hartree equation. The paper of N. Basharat, Y. Hu and S. Zheng elaborates the blowup rate for solutions of rotational NLS near the ground state profile. For instability of solitons, L. Farah, J. Holmer and S. Roudenko give simplified approach in the proof of the case for gKdV and Zakharov-Kuznetsov equations.

The stabilization of generalized Benjamin-Ono equations for water wave model is studied by C. Flores, S. Oh and D. Smith. K. Yamazaki discusses the global regularity for MHD in two dimensions. D. Li and X. Zhang obtain some optimal regularity estimates for incompressible Euler equations. For the conormal derivative problem, D. Cao, T. Mengesha and T. Phan obtain Calderón-Zygmund type regularity estimates for a class of elliptic systems with singular, degenerated coefficients. The paper of Y. Shao and C. Wang proves certain local and global regularity results for harmonic map heat flow on manifolds with conic singularities. R. Denlinger presents a short ‘deterministic’ approach to the proof of a virial type estimate concerning the derivation of the Boltzmann’s equation from a large system of interacting particles. All papers are peer-reviewed.

Recent two decades have seen rapidly growing interest in the study of the long time dynamics for dispersive evolutionary waves and fluid flows. These two broad areas might share some common mathematical challenges. For instance, how the large data theory and low regularity theory for equations of dispersive type are connected to the equations of fluids models, where some of the same open problems remain mysteries? It has become crucial to bring together researchers in the emerging areas to share, discuss and comprehend the fascinating and meanwhile hidden structures of the equations under investigation. They might address problems in connection with the central questions around regularity, long time behavior, scattering and resonance, and the spectral theory behind for the associated linear and nonlinear operators, existence of solitary waves, orbital and asymptotic stability, vorticity and wave turbulence, and singularity formation. We believe that the present proceedings with *Contemporary Mathematics* would fit in such a mission.

The volume might provide researchers a valuable reference on relevant problems in Analysis and PDEs. It might also serve as good and inspirational literature for graduate students and young mathematicians working in the area. The co-editors are content with the event that were well-attended, and especially delighted with the outcome for which it has come to fruition from the contributions and efforts.

We thank the American Mathematical Society for coordinating the special sessions at the JMM conference in Atlanta. We thank the AMS Editorial for sponsoring the publication of this special volume. We thank all the speakers, authors and participants for their enthusiasm and support, who have made the event possible and memorable.

Editors: Marius Beceanu, Jerry Bona, Geng Chen, Tuoc V. Phan, Avy Soffer

Principal Editor: Shijun Zheng

Combined List of Speakers

The underlying theme of the invited talks concentrated on latest development in the areas of analysis, evolutionary partial differential equations and mathematical physics that mainly arise in quantum mechanics, general relativity and fluid dynamics. The theory and the methods showcase recent trends and perspectives of the active program.

Here are some links to the two AMS special sessions at the JMM.

https://jointmathematicsmeetings.org/jmm2017/2180_program_ss3.html

https://jointmathematicsmeetings.org/jmm2017/2180_program_ss39.html

The following are name lists of the organizers and the speakers.

Two special sessions' organizers: Shijun Zheng, Marius Beceanu and Tuoc Van Phan; Xiang Xu, Geng Chen, and Ronghua Pan

Speakers at **Spectral Calculus and Quasilinear PDEs:**

Benjamin Harrop-Griffiths, New York University
Sung-Jin Oh, Korea Institute for Advanced Study
Jerry Bona, University of Illinois at Chicago
Jason Murphy, UC Berkeley
Zhiwu Lin, Georgia Institute of Technology
Xiaoyi Zhang, University of Iowa
Tadele Mengesha, University of Tennessee, Knoxville
Daniele Garrisi, Inha University
Hao Jia, Institute for Advanced Study
Gigliola Staffilani, Massachusetts Institute of Technology
Dong Li, University of British Columbia
Svetlana Roudenko, George Washington University
Casey Jao, UC Berkeley

Speakers at **PDE Analysis on Fluid Flows:**

Tao Huang, New York University, Shanghai
Changyou Wang, Purdue University
Wujun Zhang, Rutgers University
Tiziana Giorgi, New Mexico State University
Sookyung Joo, Old Dominion University
Andres Contreras, New Mexico State University
Geng Chen, University of Kansas

Michele Coti Zelati, University of Maryland
Mikhail Feldman, University of Wisconsin-Madison
Charis Tsikkou, West Virginia University
Dehua Wang, University of Pittsburgh
Qingtian Zhang, University of California, Davis
Kun Zhao, Tulane University
Xiaoming Wang, Florida State University
Oleksandr Misiats, Courant Institute, NYU
Qingshan Chen, Clemson University

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- 722 **Yves Aubry, Everett W. Howe, and Christophe Ritzenthaler, Editors**, *Arithmetic Geometry: Computation and Applications*, 2019
- 720 **Alexandre Girouard, Editor**, *Spectral Theory and Applications*, 2018
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This volume contains the proceedings of the AMS Special Session on Spectral Calculus and Quasilinear Partial Differential Equations and the AMS Special Session on PDE Analysis on Fluid Flows, which were held in January 2017 in Atlanta, Georgia. These two sessions shared the underlying theme of the analysis aspect of evolutionary PDEs and mathematical physics.

The articles address the latest trends and perspectives in the area of nonlinear dispersive equations and fluid flows. The topics mainly focus on using state-of-the-art methods and techniques to investigate problems of depth and richness arising in quantum mechanics, general relativity, and fluid dynamics.



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