Contemporary Mathematics

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Centre de Recherches Mathématiques Proceedings

Complex Analysis and Spectral Theory

Conference in Celebration of
Thomas Ransford's 60th Birthday
Complex Analysis and Spectral Theory
May 21–25, 2018
Laval University, Québec, Canada

H. Garth Dales
Dmitry Khavinson
Javad Mashreghi
Editors

AMS American Mathematical Society

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Preface

Spectral theory is the branch of mathematics devoted to the study of matrices and their eigenvalues, as well as their infinite-dimensional counterparts, linear operators and their spectra. Spectral theory is ubiquitous in science and engineering because so many physical phenomena, being essentially linear in nature, can be modelled using linear operators. It draws upon techniques from a variety of other areas of mathematics and leads to problems in these areas that are of interest in their own right. *Complex analysis* is the calculus of functions of a complex variable. The roots of the subject go back to the early 19th century and are associated with the names of Euler, Gauss, Cauchy, Riemann, and Weierstrass. Of particular importance are the differentiable functions, usually called analytic or holomorphic functions. They are widely used in mathematics (for example, in Fourier analysis, analytic number theory, and complex dynamics), in physics (potential theory, string theory) and in engineering (fluid dynamics, control theory and the theory of communication).

Both topics are related to numerous other domains in mathematics as well as other branches of science and engineering. For example, analytic function spaces arise in various different branches of mathematics and science. The list includes, but is not restricted to, analytical mechanics, physics, astronomy (celestial mechanics), geology (weather modeling), chemistry (reaction rates), biology, population modeling, economics (stock trends, interest rates and the market equilibrium price changes). As a matter of fact, it is hard to find a branch of analysis or applied sciences in which function spaces do not appear. Many mathematicians have studied this domain and contributed to the field and it is rather impossible to provide a list.

As another manifestation, *functional analysis* is the branch of mathematics concerned with the study of vector spaces and linear mappings acting upon them. The word "functional" refers to an operation whose argument is a function, integration, for example. Two of the most important names are Hilbert and Banach, and the central notions of the subject are named in their honor: Banach spaces (complete normed vector spaces) and Hilbert spaces (Banach spaces where the norm arises from an inner product). Hilbert spaces, which generalize the notion of Euclidean space to infinite dimensions, are of fundamental importance in many areas, including partial differential equations, quantum mechanics and signal processing. From the earliest days, researchers in functional analysis recognized the importance of studying spaces of functions, as opposed to considering just one function at a time. Together with the development of the Lebesgue integral, this led to new techniques, for example, for analyzing the behavior of analytic functions at the boundary of their domain and for proving the existence of analytic functions with

certain properties, hitherto difficult or impossible to construct. In turn, complex analysis repaid its debt to functional analysis by providing methods for defining functions of operators, for example, the square root or the logarithm of an operator or a matrix.

There are many other connections, and in the century that has followed this has become a vast domain of research. In recent years, there has been a tremendous amount of work on reproducing kernel Hilbert spaces of analytic functions, on the operators acting on them, as well as on applications in physics and engineering which arise from pure topics like interpolation and sampling.

In this conference, more than thirty analysts, some up-and-coming, others well-established, and from Europe and North America, were invited. Many different topics in complex analysis—operator theory, matrix analysis, spectral theory, functional analysis, and approximation theory—were discussed during the invited talks. This lively meeting certainly strengthened our understanding of the subjects, how far the applications range, how much is known, and how much is still unknown. The goal of our gathering was to discuss a number of fundamental open problems on Hilbert and Banach spaces of analytic functions and the new ideas that have been developed as well as the recent progress that has been made. We believe this event was worthwhile, since the ideas involved were of widespread interest in the mathematical analysis community.

In this conference, we also celebrated the 60th birthday of Thomas Ransford. Thomas Ransford is Professor in the Département de mathématiques et de statistique of Université Laval and Canada Research Chair in Spectral Theory and Complex Analysis. He obtained his Ph.D. at Cambridge in 1984, as a student of the late Graham Allan, and was awarded an Sc.D. by Cambridge in 1999. Before coming to Québec in 1993, he held teaching positions at Leeds and Cambridge. He has also held visiting positions at Ann Arbor, Bordeaux, Brown, Lille, Marseille, Oxford, and UCLA.

Ransford's research is primarily in complex analysis and spectral theory, though he has also worked in potential theory, dynamical systems and probability. He has over a hundred research publications to his name, many written in collaboration with researchers from around the world (over sixty at last count). He is also the author of two books, both published by Cambridge University Press: *Potential Theory in the Complex Plane* (written sole) and *A Primer on the Dirichlet Space* (co-authored with O. El-Fallah, K. Kellay, and J. Mashreghi). Ransford has presented over eighty invited talks at national and international conferences. Ransford is or has been a member of the editorial boards of ten different journals, including the CMS and LMS journals. He has also served on numerous selection committees for grants, fellowships, and prizes, including grant selection committees of NSERC and FRQNT.

A major aspect of Ransford's research career has been the direction of students at all levels. He has supervised 10 doctoral students, 25 master's students, and 14 postdoctoral fellows, in addition to co-directing 10 other graduate students. Many of these students have continued in academia: 22 of them now hold permanent positions at universities, and five have postdoctoral positions. Of the others, eight are now college instructors, eight others work in the financial sector, and four are computer programmers. In addition to graduate direction, Ransford has also supervised 24 summer undergraduate research projects, eight of which have given

rise to publications. In the course of his career, Ransford has taught undergraduate and graduate courses in over twenty different subjects. He has received the teaching award "professeur étoile" from the Faculté des sciences et de génie of Université Laval ten times and was voted "professeur méritant en mathématiques et statistique" at the Gala du Mérite Étudiant four times.

Mathematics runs in the family. Ransford's wife, Line Baribeau, is a mathematician, and they have published six papers together. His eldest son, Julian, is also a mathematician.

H. Garth Dales
Dmitry Khavinson
Javad Mashreghi

List of Invited Speakers

(1) W. Arendt, Ulm University, Germany
(2) R. M. Aron, Kent State University, USA
(3) C. Bénéteau, University of South Florida, USA
(4) A. Bourhim, University of Syracuse, USA
(5) I. Chalendar, Université Paris-Est Marne-la-Vallée, France
(6) J. Cima, University of North Carolina, USA
(7) C. Costara, Ovidius University, Romania
(8) H. G. Dales, Lancaster University, UK
(9) O. El-Fallah, Université Mohammed V, Morocco
(10) J. Esterle, Université de Bordeaux, France
(11) B. Forrest, University of Waterloo, Canada
(12) M. Fortier Bourque, University of Toronto, Canada
(13) R. Fournier, Université de Montréal, Canada
(14) E. Fricain, Université Lille 1, France
(15) J. Galé, University of Zaragoza, Spain
(16) P. M. Gauthier, Université de Montréal, Canada
(17) P. Gorkin, Bucknell University, USA
(18) D. Guillot, University of Delaware, USA
(19) D. Jakobson, McGill University, Canada
(20) K. Kellay, Université de Bordeaux, France
(21) D. Khavinson, University of South Florida, USA
(22) D. Kinzebulatov, Université Laval, Canada
(23) L. Kosinski, Jagiellonian University, Poland
(24) N. Levenberg, Indiana University, USA
(25) L. Marcoux, University of Waterloo, Canada
(26) J. Mashreghi, Université Laval, Canada
(27) M. Mbekhta, Université Lille 1, France
(28) A. G. O'Farrell, Maynooth University, Ireland
(29) S. Pouliasis, Aristotle University of Thessaloniki, Greece
(30) W. T. Ross, University of Richmond, USA
(31) M. Roy, York University, Canada
(32) E. Strouse, Université de Bordeaux, France
(33) M. C. White, Newcastle University, UK
(34) M. Younsi, University of Hawaii at Manoa, USA
(35) N. Zorboska, University of Manitoba, Canada
(36) W. Zwonek, Jagiellonian University, Poland

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733 **Peter Kuchment and Evgeny Semenov, Editors,** Functional Analysis and Geometry, 2019

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