Israel Moiseevich Gelfand, a mathematician compared by Henri Cartan to Poincaré and Hilbert, was born on September 2, 1913 in the small town of Okny (later Red Okny) near Odessa in Ukraine and died in New Brunswick, New Jersey, USA on October 5, 2009.

Nobody guided young Gelfand in his studies. He attended the only school in town and his mathematics teacher could offer him nothing except encouragement. And this was very important. In Gelfand’s own words: “Encouraging students is teacher’s most important job.” In 1923 the family moved to another place and Gelfand entered a vocational school for chemistry lab technicians. However, he was expelled in the ninth grade as a son of a “bourgeois” (“netrudovoi element” in soviet parlance) — his father was a mill manager. After that Gelfand, who was sixteen and a half at the time, decided to go to Moscow where he had some distant relatives.

Until his move to Moscow in 1930, Gelfand lived in total mathematical isolation. The only books available to him were several secondary school and community college textbooks. The most advanced among them claimed, for instance, that there are three types of functions: analytic, those defined by formulas; empirical, those defined by tables; and also cor relational functions. At the age of twelve Gelfand understood, on his own, that some problems in geometry could not be solved algebraically. About the same time, he also compiled a table of ratios of the length of the chord to the length of the arc. It was only much later that he learned that he had, in fact, rediscovered trigonometric tables.

From that period of Gelfand’s life came his Mozartian style of doing mathematics and his strong belief in the unity and harmony of mathematics (including applied mathematics) — the unity determined not by rigid and loudly proclaimed programs but rather by invisible and sometimes hidden connections between seemingly different areas. Gelfand described his school years and mathematical studies in an interview published in Quantum, a science magazine for high school students [1]. In Gelfand’s own words: “It is my deep conviction that mathematical ability in most future professional mathematicians appears... when they are thirteen to sixteen years old..... This period formed my style of doing mathematics. I studied different subjects but the artistic form of mathematics that took root at that time became the basis of my taste in choosing problems that continue to attract me to this day. Without understanding this motivation it is impossible, I think, to make heads and tails of the seemingly illogical ways I was working and of the choice of subjects in my research. Because of this motivating force, however, they actually come together sequentially and logically.”

This is a shortened version of a paper by V. Retakh to be published in the Notices of the Amer. Math. Soc. We are grateful to the A.M.S. for their kind permission to publish this text in MMJ.
The interview also shows how a small provincial boy was jumping over centuries in his mathematical discoveries. At the age of fifteen Gelfand learned of the power series expansion for the sine function. He described that moment in the *Quantum* interview: “Before this I thought there were two types of mathematics, algebraic and geometric. . . . When I discovered that the sine function can be expressed algebraically as a power series, the barriers came tumbling down, and mathematics reunited. To this day I see various branches of mathematics, together with mathematical physics, as parts of a whole.”

During his first years in Moscow, Gelfand couldn’t find a permanent position and lived on earnings from occasional part-time jobs. One of those jobs was at the checkout counter at the main state library (the Lenin Library). That job gave Gelfand a rare opportunity to talk to mathematics students from the Moscow University. He also started to attend various university seminars where he found himself under intense psychological stress: new breezes were blowing in mathematics with new demands for rigorous proofs. It was so different from his home-made experiments and his romantic views of mathematics. He also learned that none of his early years’ discoveries were actually new.

Just as his abrupt expulsion from school, the next twist in Gelfand’s fate was also one of many paradoxes of soviet life. On the one hand, as a son of a bourgeois he was barred from being an undergraduate university student. On the other hand, at the age of eighteen he was able to get a teaching position at one of newly created technical colleges and, at the age of just nineteen, to become a PhD student at the Moscow University. These paradoxes of the soviet system had a simple explanation: the state needed knowledgeable instructors to educate its future “engineers and scientists of proletarian origin.” However, the soviet beaurocratic machine was not sufficiently developed at that time to be able to strictly regulate graduate schools. As a result, there was a real chance for a talented young person to enter a PhD program without having a college or even a high school degree.

At the beginning of his career Gelfand was influenced by several Moscow mathematicians, especially by his thesis adviser A.N. Kolmogorov. In the *Quantum* interview, Gelfand mentioned that, among other things, he has learned from Kolmogorov “that a true mathematician must be a natural philosopher.” In 1935 Gelfand defended his PhD thesis and in 1940 obtained the (higher) degree of Doctor of Science. In 1933 he began teaching at the Moscow University. In 1943 he became a full professor and started his famous seminar. In 1952 he lost his university position temporarily during Stalin’s anticosmopolitan (in fact, antisemitic) campaign. He was allowed, however, to continue running the seminar. (Gelfand was eventually fired from the Math Department of the Moscow University in 1968 after signing a letter of support for a dissident mathematician Esenin-Volpin and refusing to withdraw his signature.) For some time, Gelfand had a position at Steklov Mathematical Institute. In 1950–1960’s he also had a job at the Institute for Applied Mathematics where he was involved in secret military linked projects, soviet analogues of the Manhattan project. Andrey Sakharov mentioned his work with Gelfand in [2].

In 1953 Gelfand was elected Corresponding Member of the Academy of Sciences of the USSR (an important title in the soviet hierarchy). That was the time following Stalin’s death and it was some political uncertainty that made Gelfand’s
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election possible. The communist party quickly regained its political authority, and antisemitism soon became part of the soviet system once again. As a result, Gelfand was not elected a full member of the soviet Academy of Sciences until 1984. By that time, he had already been a foreign member of academies of a number of European countries.

In 1989 Gelfand moved to the United States. Soon after that he became a professor at Rutgers University, where he worked for the rest of his life.

There are various descriptions of Gelfand’s multi-facet research, his way of doing mathematics, and of his interactions with other people. One well known mathematician told me that having read all the definitions in a paper by Gelfand he was able to prove all the theorems there without looking into the proofs given in the paper. In this regard, Gelfand himself liked to quote the mathematician A.Ya. Povzner who once remarked: “Gelfand cannot prove hard theorems. He just turns any theorem into an easy one.” That “simplicity” of Gelfand’s papers was usually a result of hard work, mostly invisible to the reader, that often involved long computations and a thorough analysis of various carefully selected key examples.

One cannot write about Gelfand without mentioning his legendary seminar which started in 1943 and continued until his death. Gelfand considered the seminar as one of his most important creations. It is hard to describe the seminar in a few words: it was a “mathematical stock exchange,” a breeding ground for young scientists, a demonstration how to think about mathematics, a one-man show, and much more. It was not about functional analysis or geometry, it was about Mathematics. Some would come to the seminar just to hear Gelfand’s jokes and paradoxes. Sometimes, the jokes were rather sharp and personal but for a young participant of the seminar to become a subject of Gelfand’s joke meant to be noticed, to be knighted.

Once Gelfand asked me during a seminar if I knew anything about the string theory. “I heard several talks on the subject,” I replied. “Well,” Gelfand remarked, “some people, when asked ‘Can you play violin?’ would reply ‘I believe I can, though I have never tried.’ But more sophisticated people like you would say: ‘Sure, many times I have seen others playing.’” International contacts with the West were very restricted during the soviet time. Nevertheless, a small number of leading Western mathematicians visiting USSR (P. Deligne and R. Macpherson among them) have had a chance to speak at the Seminar more than once. I remember a talk by Lipman Bers. During his talk, Bers mentioned a theorem of Maskit and he added: “I am proud that Maskit was my former student.” Gelfand immediately interrupted Bers and remarked: one cannot say “my former student”; it sounds like saying “my former son.”

Gelfand always paid special attention to students who formed the majority of the seminar audience. From time to time he would repeat: “My seminar is for high school students, decent undergraduates, bright graduates, and outstanding professors.” The seminar was a reflection of Gelfand’s passion to teach, as he tried to teach everyone and everywhere. The list of his student includes, among others, F. Berezin, J. Bernstein, E. Dyukin, A. Goncharov, D. Kazhdan, A. Kirillov, M. Kontsevich, and A. Zelevinsky. The number of his informal students is hard to estimate.

From the early period of his life also comes Gelfand’s interest in school education, especially in education of middle and high school students living in rural areas, far
from research centers. For such students, Gelfand has founded a very successful Mathematics School by Correspondence that has functioned for more than three decades. Gelfand wrote several textbooks for the School. His university textbooks *Linear Algebra* and *Calculus of Variations* (the latter joint with S. Fomin) bear a trace of his style and personality. Gelfand was among the founders of Moscow Mathematical olympiads in 1930’s.

It is hard to describe all of Gelfand’s achievements in mathematics. He was really a universal mathematician, one of the very few who have left a powerful trace in many totally different areas of mathematics. In 1930’s he was interested in functional analysis. In 1940’s he created, almost single-handedly, the theory of Banach and $C^*$-algebras (the so called Gelfand transform). In 1950’s Gelfand and Naimark in the Soviet Union, and Harish-Chandra in USA, independently developed representation theory of semisimple groups (Gelfand–Zetlin patterns, Gelfand pair). He had important works on differential equations (Gelfand–Levitan–Marchenko method). In the 60’s he continued his work (jointly with Graev, Gindikin, and Piatetski–Shapiro) in representation theory, with applications to automorphic forms. At the same time, he was also interested in the theory of distributions, and in integral geometry. In the late 1960’s and 70’s Gelfand wrote a series of joint papers with D. Fuchs on cohomology of infinite dimensional Lie algebras (Gelfand–Fuchs cohomology), two papers with A. Kirillov on enveloping algebras (Gelfand–Kirillov dimension), several joint papers on integrable systems (Gelfand–Dickey bracket), works on quiver representations (Bernstein–Gelfand–Ponomarev reflection functors). There was a series of very influential joint papers by J. Bernstein, I. Gelfand, and S. Gelfand (BGG category $O$ and BGG resolution). In 1980’s and 90’s Gelfand initiated and developed the theory of multi-dimensional hyper-geometric functions (GKZ systems) that culminated in a well-known book by I. Gelfand, M. Kapranov, and A. Zelevinsky. Jointly with R. MacPherson, he had works on combinatorial formulas for Pontryagin classes and also on combinatorial geometry (with V. Serganova). Starting in the late 90’s, I. Gelfand and V. Retakh developed a new theory of noncommutative symmetric functions and noncommutative determinants.

The above list is certainly not complete. One may consult [4] for a more complete collection of Gelfand works.

In 1978 I. Gelfand and C.L. Siegel were the first recipients of the Wolf Prize. Gelfand had many other distinguished awards, including the Kyoto Prize (1989) and the McArthur Fellowship (1994).

References