ERRATA

ON SOLUTIONS FOR THE KADOMTSEV–PETVIASHVILI I EQUATION

J. COLLIANDER, C. E. KENIG, AND G. STAFFILANI

The authors have noticed that there is an error in the proofs of Theorem 2.1 and Theorem 2.2 of [1]. The error occurs on page 513, where the authors say “We decided not to write explicitly the estimates for its linear term because they are basically contained in Section 5.” This is incorrect, since the estimate for $\| \cdot \|_7$ is not valid, while the one for $\| \cdot \|_8$ needs to be clarified. In order to make the estimate for $\| \cdot \|_7$ valid, we need to introduce an additional norm for our initial data. We now define

$$\|v\|_{1'} = \|v\|_{H^2} + \|(1 + D_x)^{3/2} (1 + D_y)^{3/2} P_- w_0\|_{L^2}.$$  

Note that the additional term, because of the presence of the projection $P_-$, is actually controlled by $\|(1 + D_y)^{5/4} w_0\|_{L^2}$. We thus need to change the abstract to read:

Abstract. Oscillatory integral techniques are used to study the well-posedness of the KP-I equation for initial data that are small with respect to the norm of a weighted Sobolev space involving derivatives of total order no larger than 2, and, in an unweighted space of order $2 + \frac{1}{4}$ in the $y$ variable.

Correspondingly, the first sentence in the paragraph in the introduction, which comes at the beginning of page 493, should read:

In this paper we use a method involving oscillatory integrals to prove that for small initial data $u_0$ in a certain weighted Sobolev space, defined with at most two derivatives, and with $2 + \frac{1}{4}$ derivatives in the $y$ variable in $L^2$, the IVP (1) is globally well-posed.

The main results should now read, with

$$\|u_0\|_{Z'_0} = \|u_0\|_{Z_0} + \|(1 + D_x)^{3/2} (1 + D_y)^{3/2} P_- w_0\|_{L^2},$$  

identically to the corresponding statements in [1], with $Z_0$ replaced by $Z'_0$, and $Z_T$ replaced by $Z'_T$, where $Z'_T$ is defined in the same way as $Z_T$ in Definition 6.1 of [1], but with $\| \cdot \|_1$ replaced by $\| \cdot \|_{1'}$ above. We now pass to the proof of Theorem 2.1, the proof of Theorem 2.2 needing to be similarly changed. We only need to estimate the linear term in (83), and also show that, for the nonlinear term in (83) we also have control of $\sup_{0 \leq t < T} \| \cdot \|_{1'}$. To estimate $\| \cdot \|_2$, we consider the first term corresponding to $D^2_x$, and use (59) in [1] to see that this term is bounded by

$$\|D^2_x P_- w_0\|_{L^2}. $$

However, Plancherel’s theorem shows that this is bounded by $\|D^5/4 w_0\|_{L^2} \leq \|w_0\|_1 \leq \|w_0\|_{1'}$. To estimate the term corresponding to $D^2_y$, we first observe that the proof of (59)

Received September 29, 2003.

529
given in [1] actually gives the estimate:
\[ \| (D_yD_x^{-1})^{1/2} U(t) P_- Q u_0 \|_{L^2_y L^2_x} \lesssim \| u_0 \|_{L^2} \].
(59')

If we now use (59'), we see that the term corresponding to \( D_y^2 \) in the estimate of \( \| \cdot \|_7 \), is bounded by
\[ \| D_y^{3/2} D_y^{3/2} P_- u_0 \|_{L^2} \lesssim \| u_0 \|_1 \].
To estimate \( \| \cdot \|_8 \), we again use (59'), and see that \( \| \cdot \|_8 \) is controlled by
\[ \| \partial_x (1 + D_x)^{\sigma_1 + 1/2} (1 + D_y)^{\gamma_1 - 1/2} P_- u_0 \|_{L^2} , \]
which, since \( \sigma_1 > \frac{3}{2}, \gamma_1 > \frac{1}{2} \), but both are close to \( \frac{3}{2}, \frac{1}{2} \), respectively, is bounded by (using Plancherel and taking advantage of \( P_- \))
\[ \| (1 + D_y)^{3/2} (1 + D_y)^{1/2} \|_{L^2} \lesssim \| u_0 \|_1 \leq \| u_0 \|_8 \].
Finally, in order to control \( \sup_{0 \leq t \leq T} \| \cdot \|_{L^1_x} \), for the nonlinear term in (83) of [1], we only need to control the part of the norm corresponding to \( (1 + D_y)^{3/2} (1 + D_y)^{1/2} P_- \). In order to do this, we observe that (59) yields, by duality, the estimate
\[ \sup_{0 \leq t \leq T} \left\| D_y^{1/2} \int_0^t U(t - t') \partial_x (u^3) dt' \right\|_{L^2_y} \lesssim \| h \|_{L^2_x L^2} . \]
(59'')

Using (59''), we see that since we need to estimate
\[ \left\| (1 + D_x)^{3/2} (1 + D_y)^{3/2} P_- \int_0^t U(t - t') Q \partial_x (w^2) dt \right\|_{L^2_y} , \]
and by Plancherel and the presence of \( P_- \), this is controlled by
\[ \left\| D_y^{3/2} (1 + D_y)^2 P_- \int_0^t U(t - t') Q \partial_x (w^2) dt \right\|_{L^2_y} , \]
which, by (59''), is controlled by
\[ \| \partial_x (1 + D_y)^2 (w^2) \|_{L^1_x L^2} , \]
which is already bounded by the last line of page 513 of [1], and an estimate in page 514 of [1]. With these changes the rest of the proof of Theorems 2.1 and 2.2 of [1] proceeds in the same way as in [1].

The authors have also noticed that Remark 6.6 in [1] is in error. In particular, the statement that for the initial value problem for modified KP-I, (93) in [1], the smallness assumption on the initial data can be removed, is erroneous, since the scaling argument does not apply. Thus, Theorem 6.7 is not known to hold for large initial data. In a forthcoming paper [2] it will be shown that (93) is in fact locally well-posed, by Picard iteration, for small data, in the space \( \{ f: f \in L^2, D_x^{3/2} f \in L^2, (D_y D_x^{-1})^{3/2} \} \).

REFERENCES


J. C.: UNIVERSITY OF TORONTO. E-mail address: colliand@math.toronto.edu

C. K.: UNIVERSITY OF CHICAGO. E-mail address: cek@math.uchicago.edu

G. S.: BROWN UNIVERSITY AND STANFORD UNIVERSITY.
E-mail address: gigliola@math.brown.edu