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# JOURNAL

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## IN MEMORIAM

It is with great sadness that we note the passing of Oded Schramm, who died on September 1, 2008, in a hiking accident. Oded had been an editor of the *Journal of the American Mathematical Society* since late 2007. Even in this short time, the *Journal* profited greatly from his mathematical vision and editorial wisdom.

Oded was born in Israel in 1961 and obtained his Ph.D. from Princeton in 1990 under the direction of William Thurston. He held positions at the University of California, San Diego, the Weizmann Institute, and (since 1999) at Microsoft.

Oded's initial research focused on packing problems but soon branched out to encompass percolation and other random structures, conformal geometry, and property testing. His most celebrated achievement is the discovery of the *Schramm-Loewner equation* in 2000, a stochastic equation that governs the behavior of a wide variety of random processes, such as the loop-erased random walk, the Brownian frontier, and percolation on the triangular lattice. Exploring the further connections of this equation to other random processes and to conformal field theory remains a major area of active research. For this and other work, Oded was honored with numerous awards, including the Salem, Loéve, Poincaré, Pólya, and Ostrowski prizes, the Clay Research Award, and a foreign membership in the Royal Swedish Academy of Sciences.

Oded is survived by his wife Avivit and two children.

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