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ERRATA TO "CONTRAVARIANT FORMS ON WHITTAKER MODULES"

ADAM BROWN AND ANNA ROMANOV

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Abstract. Here we make corrections to address a false statement by Brown and Romanov [Proc. Amer. Math. Soc. 149 (2021), pp. 37–52].

As stated, Lemma 3.10 of [BR21], and the following equations (3.7) and (3.8), are false. The remaining results of [BR21] can be recovered with the following corrections.

(1) Remove lines 14–29 on page 43. This content consists of the set-up for, statement of, proof, and immediate consequences of [BR21, Lemma 3.10].
(2) Replace the proof of part (a) of [BR21, Lemma 3.12] with the following proof, which no longer refers to Lemma 3.10 or equation (3.8) (with all of the notation described in [BR21]):

Proof. Let t_ρ be the algebra automorphism of $U(\mathfrak{h})$ induced by the ρ-twisting map $h \mapsto h - \rho(h)$ for $h \in \mathfrak{h}$. The composition of the Harish-Chandra homomorphism p_0 (Definition 3.6, Remark 3.7) with t_ρ provides an algebra isomorphism

$$t_\rho \circ p_0 : Z(\mathfrak{g}) \xrightarrow{\sim} S(\mathfrak{h})^W$$

[Bou05, Ch. VIII, §8.5, Thm. 2].

The ideal S is generated by $S(\mathfrak{h})^W_+$, so any element of S can be expressed as a sum of elements of the form

$$ht_\rho(p_0(z))$$

for various $z \in Z(\mathfrak{g})$ and $h \in S(\mathfrak{h}) = U(\mathfrak{h})$. Our first step in the proof is to show that any element of the form $ht_\rho(p_0(z))$ satisfies (a).

Any $u \in U(\mathfrak{g})$ is a linear combination of Poincaré–Birkhoff–Witt basis elements of the form

$$y^{\overline{I}} h^J x^K := y_{\alpha_n}^{i_n} \cdots y_{\alpha_1}^{i_1} h_{\alpha_1}^{j_1} \cdots h_{\alpha_r}^{j_r} x_{\alpha_1}^{k_1} \cdots x_{\alpha_n}^{k_n}.$$

Hence we can express $u \in U(\mathfrak{g})$ as a sum

$$u = p_0(u) + \sum_{I,J,K} a_{I,J,K}(u) y^{\overline{I}} h^J x^K, \tag{3.11}$$

where $a_{I,J,K}(u) \in \mathbb{C}$ and $a_{I,J,K}(u) = 0$ if $I = J = (0, \ldots, 0)$. By applying p_η to (3.11) and using equation (3.3), we obtain

$$p_\eta(u) = p_0(u) + \sum_{I,J,K} a_{I,J,K}(u) \eta(x^K x^I) h^J. \tag{3.12}$$

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Because the composition $t_\rho \circ p_0 : Z(\mathfrak{g}) \to S(\mathfrak{h})^W$ induces an isomorphism between the corresponding graded objects (with the grading of $Z(\mathfrak{g})$ induced by the natural filtration of $U(\mathfrak{g})$ by $\mathfrak{g}$) [Bou05, Ch.VIII §8.5 proof of Thm. 2], for $z \in Z(\mathfrak{g})$, we have

$$\deg(h^J) < \deg(p_0(z)) \text{ for all } J \text{ such that } a_{I,J,K}(z) \neq 0. \tag{3.13}$$

Hence the image of $z \in Z(\mathfrak{g})$ under the η-twisted Harish-Chandra projection p_η and the image of z under the Harish-Chandra homomorphism p_0 agree up to lower degree terms. To increase readability in the arguments below, we will introduce some notation to describe this phenomenon in general. Write LDP for either an element in $U(\mathfrak{h})$ with degree strictly lower than the element immediately preceding it in an expression, or zero.[1] For example, by (3.13), we can rewrite (3.12) as

$$p_\eta(z) = p_0(z) + LDP. \tag{3.14}$$

Similarly, for all $h \in U(\mathfrak{h})$, $t_\rho(h) = h+\text{LDP}$ and $p_\eta(hz) = hp_0(z)+\text{LDP}$. Therefore, we have

$$ht_\rho(p_0(z)) = hp_0(z) + LDP = p_\eta(hz) + LDP. \tag{3.15}$$

By the linearity of p_η, we have

$$ht_\rho(p_0(z)) = p_\eta(h(z - \chi(z))) + LDP, \tag{3.16}$$

for $z \in Z(\mathfrak{g})$ with degree greater than or equal to one. We conclude that any element of S which is equal to $ht_\rho(p_0(z))$ for some $h \in U(\mathfrak{h})$ and $z \in Z(\mathfrak{g})$ satisfies (a).

An arbitrary element $s \in S$ is a sum of elements of the form $ht_\rho(p_0(z))$ for various $h \in U(\mathfrak{h})$ and $z \in Z(\mathfrak{g})$, so by the linearity of p_η, there exists $k \in U(\mathfrak{g}) \ker \chi$ such that

$$s = p_\eta(k) + LDP.$$

This proves (a). □

Acknowledgment

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References

[Bou05] Nicolas Bourbaki, *Lie groups and Lie algebras. Chapters 7–9*, Elements of Mathematics (Berlin), Springer-Verlag, Berlin, 2005. Translated from the 1975 and 1982 French originals by Andrew Pressley. MR2109105

[BR21] Adam Brown and Anna Romanov, *Contravariant forms on Whittaker modules*, Proc. Amer. Math. Soc. **149** (2021), no. 1, 37–52, DOI 10.1090/proc/15205. MR4172584

Mathematical Institute, University of Oxford, Andrew Wiles Building, Radcliffe Observatory Quarter (550), Woodstock Road, Oxford, UK OX2 6GG

Department of Mathematics, University of New South Wales, Sydney NSW 2052, Australia

[1] Here LDP stands for "lower degree polynomial."