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CORRIGENDUM TO "INTERMEDIATE C*-ALGEBRAS OF CARTAN EMBEDDINGS"

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The statement of [1, Theorem 4.2] is correct, however there is an error in the proof. The penultimate sentence of the proof claims that the compactly supported function $m_{h_i}(f)$ in $C_c(\Sigma, G)$, which vanishes off the open subgroupoid $H \subseteq G$, must lie in $C_c(\Sigma_H, H)$. If H were closed, this statement would be correct, but it is not always true in the context of [1]. We thank Pradyut Karamakar for helping us find the error.

We provide a correct proof of Theorem 4.2. Throughout we will use the notation and terminology established in [1].

Theorem (cf. [1, Theorem 4.2])**.** *Let $\Sigma \to G$ be a twist with G an étale, Hausdorff, amenable groupoid. Let H be an open subgroupoid of G. Then A_H is equal to the canonical copy of $C^*_r(\Sigma_H; H)$ within $C^*_r(\Sigma; G)$.*

Proof. By the same reasoning as presented in [1], there is a net of positive-type functions $h_i \in C_c(G)$ converging uniformly to 1 on compact subsets of G, with $\sup_{\gamma \in G} |h_i(\gamma)| \leq 1$. The multipliers

$$\begin{aligned} m_{h_i} \colon C_c(\Sigma; G) &\to C_c(\Sigma; G) \\ f &\mapsto h_i f \end{aligned}$$

extend to completely positive maps on $C^*_r(\Sigma; G)$ such that for all $f \in C^*_r(\Sigma; G)$,

- the net $(m_{h_i}(f))_i$ converges to f in $C^*_r(\Sigma; G)$; and
- $m_{h_i}(f) \in C_c(\Sigma; G)$.

Take any $f \in A_H$. We wish to show that $f \in C^*_r(\Sigma_H; H)$. As $(m_{h_i}(f))_i$ converges to f in the reduced norm, it suffices to show that each $m_{h_i}(f)$ is in $C^*_r(\Sigma_H; H)$. Further, as $m_{h_i}(f)$ lies in $C_c(\Sigma; G)$ and vanishes off H, we may assume that f lies in $C_c(\Sigma; G)$. Let K be a compact subset of G outside of which f vanishes. Since G is étale, we may find finitely many open bisections $U_1, U_2, \ldots, U_n$, whose union contains K. Let $\varphi_1, \ldots, \varphi_n$ be a partition of unity for K subordinate to the open cover $U_1, \ldots, U_n$. That is, choose $\varphi_1, \ldots, \varphi_n$ so that each φ_i is a continuous, nonnegative real-valued function on G, vanishing off U_i, and such that for each

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$\gamma \in K$,

$$\sum_{i=1}^{n} \varphi_i(\gamma) = 1.$$

Hence

$$f = \sum_{i=1}^{n} f\varphi_i,$$

where $\varphi_i f$ is the pointwise product. Therefore, it is enough to show that $\varphi_i f \in C_r^*(\Sigma_H; H)$ for each i. Note that $\varphi_i f$ vanishes off the bisection $U_i \cap H$. Thus we may assume without loss of generality that our originally chosen f vanishes outside some open bisection U, with $U \subseteq H$.

Set $V = s(U)$, and let $(u_j)_j$ be an approximate unit for $C_0(V)$ contained in $C_c(V)$. Observe that, for each j, we have $(f * u_j)(\gamma) = f(\gamma)u_j(s(\gamma))$. As s establishes a homeomorphism between U and V, it follows that

$$\|f * u_j - f\|_\infty = \|(u_j \circ s)f - f\|_\infty \longrightarrow 0.$$

As U is a bisection it follows that $f * u_j \to f$ in reduced norm also, see e.g. [2, Theorem 11.1.11]. Noting that $f * u_j$ is compactly supported in $U \subseteq H$ it follows that

$$f * u_j \in C_c(\Sigma_H; H) \subseteq C_r^*(\Sigma_H; H),$$

and hence $f \in C_r^*(\Sigma_H; H)$. □

References

[1] Jonathan H. Brown, Ruy Exel, Adam H. Fuller, David R. Pitts, and Sarah A. Reznikoff, *Intermediate C^*-algebras of Cartan embeddings*, Proc. Amer. Math. Soc. Ser. B **8** (2021), 27–41, DOI 10.1090/bproc/66. MR4199728

[2] Aidan Sims, Gábor Szabó, and Dana Williams, *Operator algebras and dynamics: groupoids, crossed products, and Rokhlin dimension*, Advanced Courses in Mathematics. CRM Barcelona, Birkhäuser/Springer, Cham, [2020] ©2020. Lecture notes from the Advanced Course held at Centre de Recerca Matemàtica (CRM) Barcelona, March 13–17, 2017; Edited by Francesc Perera, DOI 10.1007/978-3-030-39713-5. MR4321941

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