n_1 + 1, \ldots, \text{then the group generated by } i_1 \text{ and } H \text{ contains operators of order } p^2 \text{ and the remarks in regard to additional groups apply only to the remaining numbers and to the in-variant operators of } H \text{ which are not commutators. As } i_1 \text{ and its conjugates cannot give rise to any group of order } p^m \text{ when } p \text{ is less than some one of the numbers } n_1 + 1, n_2 + 1, \ldots, \text{all the groups of this order which contain } H \text{ can be readily obtained by the above considerations. It may be observed that this includes all the groups of order } p^m \text{ in which every operator is of order } p \text{ whenever } m < 5, \text{since every group of order } p^4 \text{ contains an abelian subgroup of order } p^3.

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A CLASS OF SIMPLY TRANSITIVE LINEAR GROUPS.

BY PROFESSOR L. E. DICKSON.

1. In the study of the group defined for any given field by the multiplication table of any given finite group,* it is necessary to discuss the types of simply transitive linear homogeneous groups \( G \) whose transformations can be given the form

\[
\begin{align*}
\xi_1' &= \gamma_1 \xi_1, \\
\xi_2' &= \gamma_2 \xi_2 + \gamma_1 \xi_1, \\
\xi_3' &= \gamma_3 \xi_3 + a \xi_2 + \gamma_1 \xi_1, \\
\xi_4' &= \gamma_4 \xi_4 + \beta \xi_2 + \gamma_3 \xi_3 + \gamma_1 \xi_1, \\
\xi_5' &= \gamma_5 \xi_5 + \gamma_4 \xi_4 + \alpha \xi_3 + \gamma_3 \xi_3 + \gamma_1 \xi_1, \\
&\ldots.
\end{align*}
\]

Here \( \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \ldots \) are the independent parameters, while \( a, \beta, \gamma_1, \lambda, \ldots \) are linear homogeneous functions of the \( \gamma_1 \). Burnside† was led to the erroneous conclusion that every such group \( G \) is an abelian group. He first concludes that the expression for \( \xi'_1 \) contains only the parameters \( \gamma_1, \ldots, \gamma_4 \) and contains \( \gamma_5 \) only in the first term \( \gamma_5 \xi_1 \). That this result need not be true is shown by a consideration of the simply transitive group of quaternary transformations

\[
\begin{align*}
\xi_1' &= \gamma_1 \xi_1, \\
\xi_2' &= \gamma_2 \xi_2 + \gamma_1 \xi_1, \\
\xi_3' &= \gamma_3 \xi_3 + a \xi_2 + \gamma_1 \xi_1, \\
\xi_4' &= \gamma_4 \xi_4 + \beta \xi_2 + \gamma_3 \xi_3 + \gamma_1 \xi_1, \\
\xi_5' &= \gamma_5 \xi_5 + \gamma_4 \xi_4 + \frac{a_1}{a_2} \xi_3 + \gamma_3 \xi_3 + \gamma_1 \xi_1, \\
&\ldots.
\end{align*}
\]

where \( a = a_1 \gamma_1 + a_2 \gamma_2 + a_3 \gamma_3 \), \( a_i \neq 0 \). Let \( Y_i \) be the infinitesimal transformation obtained by setting \( \gamma_i = \delta t_i \), \( \gamma_i = 0 \) \((j = 1, 2, 3, 4; j \neq i)\). Then

\[
Y_1 = \sum_{i=1}^{4} \xi_i \frac{\partial f}{\partial \xi_i}, \quad Y_2 = \xi_1 \frac{\partial f}{\partial \xi_2} + a_2 \xi_2 \frac{\partial f}{\partial \xi_2} - a_3 \xi_3 \frac{\partial f}{\partial \xi_3},
\]

\[
Y_s = (\xi_1 + a_2 \xi_2) \frac{\partial f}{\partial \xi_3} - a_3 \xi_3 \frac{\partial f}{\partial \xi_3},
\]

\[
Y_4 = a_4 \xi_4 \frac{\partial f}{\partial \xi_4} + (\xi_1 - a_2 \xi_2) \frac{\partial f}{\partial \xi_4}, \quad (Y_2 Y_3) = a_3 Y_3 - a_4 Y_4,
\]

\[
(Y_2 Y_4) = a_4 Y_4 - a_2 Y_2, \quad (Y_1 Y_2) = 0, \quad (Y_1 Y_3) = 0.
\]

But the desired result can always be reached by applying a suitable transformation on the variables \( \xi \), and the cogradient transformation on the parameters \( \gamma \) (see §§ 4, 5 below). Taking \( G \) in this reduced form, Burnside attempts to prove by induction that \( G \) is abelian. He supposes that the first \( t - 1 \) equations of \( G \) define an abelian group and concludes, from the fact that \( G \) is its own parameter group, that the subgroup of \( G \) generated by the infinitesimal operations \( Y_1, Y_2, \ldots, Y_{t-1} \) corresponding to \( \gamma_1, \gamma_2, \ldots, \gamma_{t-1} \) is abelian. The invalidity of the conclusion is shown by an example. The transformations

\[
(3) \quad \xi_1' = \gamma_1 \xi_1, \quad \xi_2' = \gamma_2 \xi_1 + \gamma_1 \xi_2, \quad \xi_3' = \gamma_3 \xi_1 + \gamma_1 \xi_3, \quad \xi_4' = \gamma_4 \xi_1 + (b_2 \gamma_2 + b_3 \gamma_3) \xi_2 + (c_2 \gamma_2 + c_3 \gamma_3) \xi_3 + \gamma_1 \xi_4
\]

constitute a simply transitive group in reduced form, which is its own parameter group. The first three equations taken alone constitute an abelian group. But \( Y_1, Y_2, Y_3 \) do not generate a group if \( b_4 \neq c_4 \). In fact,

\[
Y_1 = \sum_{i=1}^{4} \xi_i \frac{\partial f}{\partial \xi_i}, \quad Y_2 = \xi_1 \frac{\partial f}{\partial \xi_2} + (b_2 \xi_2 + c_2 \xi_3) \frac{\partial f}{\partial \xi_3},
\]

\[
Y_s = \xi_1 \frac{\partial f}{\partial \xi_3} + (b_3 \xi_3 + c_3 \xi_4) \frac{\partial f}{\partial \xi_4}, \quad Y_4 = \xi_1 \frac{\partial f}{\partial \xi_4},
\]

\[
(Y_1 Y_2) = 0, \quad (Y_1 Y_3) = 0, \quad (Y_1 Y_4) = (b_4 - c_4) Y_4,
\]

\[
(Y_1 Y_3) = 0 \quad (i = 1, 2, 3).
\]

Each of the two preceding examples shows that \( G \) need not be abelian.

2. For the case of one variable or the case of two variables,
the transformations (1) evidently form a simply transitive abelian group. We proceed to consider the cases $n = 3, 4, 5$.

The method applies immediately to $n$ variables, but the formulæ are complicated by the necessary use of a triple subscript notation for the coefficients. Set

$$a = \sum_{k=1}^{n} a_k g_k, \quad \beta = \sum_{k=1}^{n} b_k g_k, \quad \gamma = \sum_{k=1}^{n} c_k g_k$$

$$\lambda = \sum_{k=1}^{n} l_k g_k, \quad \mu = \sum_{k=1}^{n} m_k g_k, \quad \nu = \sum_{k=1}^{n} n_k g_k$$

where the $a_1, \ldots, a_n$ are constants for a particular group $G$. The general transformation of $G$ will be designated $T_1$ or $T_2, \ldots, T_n$. Then the product $T_1 T_2$ is of the form (1) with $\eta_1'', \eta_2'', \ldots, \eta_n''$, in place of $\eta_1, \eta_2, \ldots, \eta_n$, where

$$\eta_1'' = \eta_1 \eta_1', \quad \eta_2'' = \eta_2 \eta_2' + \eta_1 \eta_2', \quad \eta_3'' = \eta_3 \eta_1' + \alpha \eta_2' + \eta_3 \eta_2', \quad \eta_4'' = \eta_4 \eta_1' + \beta \eta_2' + \gamma \eta_3' + \eta_4 \eta_3', \quad \eta_5'' = \eta_5 \eta_1' + \lambda \eta_2' + \mu \eta_3' + \nu \eta_4' + \eta_5 \eta_4', \quad a'' = a_1 + \eta_2 \eta_1' + \eta_1 \eta_2', \quad \beta'' = \alpha \eta_1' + \gamma \eta_2' + \eta_1 \nu + \eta_2 \eta_1', \quad \lambda'' = \lambda \eta_1' + \mu \eta_2' + \nu \eta_3' + \eta_1 \mu + \eta_2 \lambda, \quad \nu'' = \nu \eta_1' + \eta_2 \nu.$$ 

The transformations (1) will form a group if, and only if, $T_1 T_2 = T_2 T_1$, where $\eta_1'', \ldots, \eta_n''$ have the values just given, while the relations

$$a'' = \sum_{k=1}^{n} a_k g_k'', \quad \beta'' = \sum_{k=1}^{n} b_k g_k'', \quad \gamma'' = \sum_{k=1}^{n} c_k g_k''$$

reduce to identities in $\eta_1$ and $\eta_2$. Upon replacing $\alpha'', \beta'', \ldots, \eta_n''$ by the above values, we find that $a_1, b_1, c_1, l_1, m_1, n_1$ are zero. For $n = 5$, the remaining conditions are

(4) $a_1 a_1 + a_1 b_1 + a_1 c_1 = 0, \quad a_1 c_1 + a_1 m_1 = 0, \quad a_1 n_1 = 0,$

(5) $c_1 a_1 + c_1 b_1 + c_1 l_1 = 0, \quad c_1 c_1 + c_1 m_1 = 0, \quad c_1 n_1 = 0,$

(6) $n_1 a_1 + n_1 b_1 + n_1 l_1 = 0, \quad n_1 c_1 + n_1 m_1 = 0, \quad n_1 n_1 = 0,$

(7) $b_1 a_1 + b_1 b_1 + b_1 l_1 = a_1 c_1, \quad b_1 c_1 + b_1 m_1 = a_1 c_1, \quad b_1 n_1 = a_1 c_1, \quad 0 = a_1 c_1.$
(8) \[
\begin{align*}
    & m_2 a_x + m_3 b_x + m_4 l_x = a_2 n_x, \quad m_4 c_x + m_5 m_x = c_4 n_x, \\
    & m_5 n_x = c_5 n_x, \quad 0 = c_5 n_x,
\end{align*}
\]
(9) \[
\begin{align*}
    & l_x a_x + l_y b_x + l_z a_y = a_2 m_x + b_x n_x, \quad l_x c_x + l_y m_x = a_2 m_x + b_x n_x, \\
    & l_y n_x = a_2 m_x + b_x n_x, \quad 0 = a_2 m_x + b_x n_x,
\end{align*}
\]
where \( k \) takes the values 2, 3, 4, 5.

3. For \( n = 3 \), the \( a_x \) are the only coefficients to be considered, and the preceding conditions reduce to \( a_2 a_x = 0 (k = 2, 3) \). Hence \( a = a_2 \gamma_1 \). Then \( \gamma_1'' \), \( \gamma_1''' \), \( \gamma_1'''' \) are symmetrical in \( \gamma_i \) and \( \gamma_i'' \), so that the group is abelian.

4. For \( n = 4 \), \( l_x, m_x, n_x \) do not occur, so that conditions are

\[
\begin{align*}
    & a_2 a_x + a_3 b_x = 0, \quad a_3 c_x = 0, \quad c_4 a_x + c_5 b_x = 0, \quad c_5 c_x = 0, \\
    & b_2 a_x + b_3 b_x = a_3 c_x, \quad b_3 c_x = a_3 c_x, \quad 0 = a_3 c_x,
\end{align*}
\]
(\( k = 2, 3, 4 \)).

Hence \( \gamma_1 = 0 \). If either \( c_4 \) or \( c_5 \) is not zero, the conditions reduce to

\[
\begin{align*}
    & a_4 = 0, \quad b_4 = a_4 = 0, \quad c_4 a_x = 0, \quad b_4 b_x = a_4 c_x.
\end{align*}
\]
If \( a_4 \neq 0 \), then \( a = a_4 \gamma_1 \), \( \beta = b_4 \gamma_1 \), \( \gamma = b_4 \gamma_1 \), and \( \gamma_1'' \), \( \gamma_1''' \), \( \gamma_1'''' \) are symmetrical in \( \gamma_i \) and \( \gamma_i'' \). The group is therefore abelian. If \( a_4 = 0 \), \( T_4 \) takes the form (3). The group \( G \) is abelian if, and only if, \( b_4 = c_4 \). If \( b_4 = c_4 \), the only "ausgezeichnete" infinitesimal transformations are the \( e Y_1 + e Y_4 \).

Let next \( c_4 = e_4 = 0 \), so that the conditions are

\[
\begin{align*}
    & a_2 a_x + a_3 b_x = 0, \quad b_2 a_x + b_3 b_x = 0 \quad (k = 2, 3, 4).
\end{align*}
\]
If \( a_4 = 0 \), then \( a_3 = b_3 = 0, b_4 a_2 = 0 \). If also \( a_2 = 0 \), \( T_4 \) is of the form (3) with \( c_2 = c_3 = 0 \). But if \( a_2 \neq 0 \), then \( a = a_4 \gamma_1 \), \( \beta = b_4 \gamma_1 \), \( \gamma = b_4 \gamma_1 \), so that the group is abelian. Finally, if \( a_4 \neq 0 \), the conditions reduce to the following:

\[
\begin{align*}
    & a_4 + b_4 = 0, \quad a_3 + a_4 b_3 = 0, \quad a_4 a_4 + a_4 b_4 = 0,
\end{align*}
\]
whence \( \beta = -a_4 a_4, \gamma = 0 \), so that \( T_4 \) is of the form (2). The group \( G \) is then not abelian (§1). To bring it to the reduced form, set

\*The group is of the type \((V')\), page 588, Lie-Scheffers, Continuierliche Gruppen.
Then $T_\eta$ becomes

$$
\xi_1' = \eta_1\xi_1, \quad \xi_2' = \eta_2\xi_2, \quad \xi_3' = \eta_3\xi_3 + \eta_1x_4, \\
\xi_4' = \eta_4\xi_4 - \frac{a_8}{a_6} (a_2\eta_5 + \xi_3') + \eta_1\xi_4.
$$

Its self-conjugate transformations are the following:

$$
\xi_1' = \eta_1'\xi_1, \quad \xi_2' = \eta_2'\xi_2, \quad \xi_3' = \eta_3'\xi_3, \quad \xi_4' = \eta_4'\xi_4 + \eta_1\xi_4.
$$

The group of transformations (1) on four variables is either abelian or else is one of the types (2) and (3), whose self-conjugate transformations form groups of exactly two parameters.

5. Let next $n = 5$. Then $n = 0$. If $a_5 \neq 0$, the last of the conditions (4) and (7) give $n = 0$, $c_5 = 0$, and the second condition (4) gives $m = 0$. Hence $\gamma = \mu = \nu = 0$. The first condition (4) gives $a_0 + a_3 + a_5 = 0$. Set

$$
x_3 = a_3\xi_3 + a_4\xi_4 + a_5\xi_5, \quad \xi_3 = a_3\eta_3 + a_4\eta_4 + a_5\eta_5.
$$

Then $x_3' = \xi_3\xi_4 + \eta_1x_4$, so that, by applying a transformation on $\xi_3', \xi_4$, and $\xi_5$ and a transformation on the parameters $\eta_3, \eta_4, \eta_5$, we obtain a transformation (1) with $a = 0$. Set

$$
x_3 = a_0\xi_3 + a_3\xi_4, \quad \xi_3 = a_0\eta_3 + a_3\eta_4.
$$

Then $x_3' = \xi_3\xi_4 + \eta_1x_4$. If $a_0 = a_3 = 0$, then $a_0a_3 = 0$ by (4), so that $a_3 = 0$. Let $a_3 = a_5 = a_4 = 0$, $c_5 \neq 0$. Then $n = 0$ by the third equation (5), so that $\tau = 0$. Also $c_4c_5a_2 + c_5^2 + c_4a_5 = 0, c_4 + c_5a = 0$ by the first and second equations (5). Set

$$
x_4 = a_4\xi_4 + a_5\xi_5, \quad \xi_4 = a_4\eta_4 + a_5\eta_5.
$$

Then $x_4' = \xi_4\xi_5 + \eta_1x_5$. Hence, by applying a transformation on $\xi_4, \xi_5$ and one on the parameters $\eta_4, \eta_5$, we obtain a transformation (1) with $\gamma = 0$. Let $a_5 = a_4 = a_3 = 0$, $c_5 = 0$. Then $c_4 = 0$ by (5). If $c_4 \neq 0$, then $n = 0$ by the third equation (5), so that $\nu = 0$. By the first and second equations (5),

$$(b_3 - c_4)a_3\eta_4 - a_4c_5\eta_5 + b_4\beta + b_5\gamma = 0, \quad b_4\nu + b_5\mu = 0.$$
Hence
\[ x'_i = \xi'_i \xi^{-1}_i + [(c_i - b_i) a_i y_i + a_i c_i y_i] \xi_i + \eta_i x_i, \]
\[ x_4 = b_4 \xi'_4 + b_6 \xi_6, \quad \xi_4 = b_4 \eta_4 + b_6 \eta_6. \]

We may therefore take \( b_5 = 0 \). Then \( b_5 = 0 \) by the first equation (7), so that \( b_5 = 0 \). Then \( m_4 = 0 \) by the second equation (8). Hence \( \eta_5 = 0 \) by the first equation (9).

Hence \( T_\eta \) becomes
\[ \xi'_i = \eta_i \xi_i, \quad \xi'_i = \eta_i \xi_i + n_i \xi_i, \quad \xi'_i = \eta_i \xi_i + m_i \xi_i + n_i \xi_i, \]
\[ \xi'_i = \eta_i \xi_i + (m_i + n_i) \xi_i, \quad \xi'_i = \eta_i \xi_i + (m_i + n_i) \xi_i + \eta_i \xi_i, \]
\[ \xi'_i = \eta_i \xi_i + (m_i + n_i) \xi_i + \eta_i \xi_i, \quad \xi'_i = \eta_i \xi_i + (m_i + n_i) \xi_i + \eta_i \xi_i. \]

Since the group is now in its reduced form, it contains the self-conjugate transformations, in which \( \gamma_i \) and \( \gamma_i' \) are arbitrary, \( \gamma_i' \neq 0 \),
\[ \xi'_i = \gamma_i \xi_i (i = 1, 2, 3, 4), \quad \xi'_i = \gamma_i \xi_i + \eta_i \xi_i. \]

6. The conditions (4)–(9) on \( T_\eta \) in its reduced form are
\[ c_i a_i = 0, \quad n_i a_i + n_i b_i = 0, \quad b_i = 0, \quad n_i c_i = 0, \quad n_i c_i = 0, \]
\[ b_i a_i = a_i c_i, \quad m_i a_i + m_i b_i = c_i n_i, \quad m_i b_i = c_i n_i, \quad m_i c_i = c_i n_i, \]
\[ m_i c_i = c_i n_i, \quad (m_i + n_i) \xi_i = (m_i + n_i) \xi_i, \quad (m_i + n_i) \xi_i = (m_i + n_i) \xi_i, \]
\[ 0 = a_i c_i + b_i n_i, \quad l_i c_i = b_i n_i, \quad l_i c_i = b_i n_i. \]

If \( n_i = 0 \), then \( b_i = c_i = 0 \), \( m_i a_i + n_i b_i = 0 \). Set
\[ x_i = n_i \xi_i + n_i \xi_i, \quad \xi_i = m_i n_i + n_i \xi_i. \]

Then \( x'_i = \xi_i \xi_i + \gamma_i x_i \). Hence by introducing \( x_i \) in place of \( \xi_i \) and \( \gamma_i \) in place of \( \eta_i \), \( T_\eta \) retains its reduced form and has \( b_i = b_i = c_i = 0 \). Then
\[ (10) \quad n_i a_i = 0, \quad m_i a_i = 0, \quad l_i a_i = m_i a_i, \quad m_i a_i = 0, \]
are the only further conditions.

If \( a_i \neq 0 \), we obtain the transformation
\[ \xi'_i = \gamma_i \xi_i, \quad \xi'_i = \gamma_i \xi_i + \gamma_i \xi_i, \quad \xi'_i = \gamma_i \xi_i + \gamma_i \xi_i, \]
\[ \xi'_i = \gamma_i \xi_i + \gamma_i \xi_i, \quad \xi'_i = \gamma_i \xi_i + \gamma_i \xi_i, \quad \xi'_i = \gamma_i \xi_i + \gamma_i \xi_i. \]
It is readily verified that these transformations form a group with the parameters $\gamma_i$, $\gamma_j$, whatever be the values of $a_i$, $t_i$, $t_n$, $t_n$, $n_i$, $n_j$. The expressions for $\gamma_i''(i = 1, 2, 3, 4)$ are symmetric in $\gamma_i$ and $\gamma_i'$, but that for $\gamma_n''$ is symmetric if, and only if, $n = t$. In the latter case only, the group is abelian. For $n = t$, a transformation will belong also to the reciprocal group if, and only if, $\gamma_n = \gamma_i = 0$. Hence the subgroup of self-conjugate transformations has three arbitrary parameters $\gamma_i, \gamma_j, \gamma_n$.

But, if $a_i = 0$, the conditions (10) become identities. Hence the transformations

$$T_i \text{ with } a_i = b_i = b_i = c_i = e_i = 0, \ n_i = 0,$$

form a group, whatever be the values of $l_i, m_i, n_i$. It is abelian if, and only if, $m_i = l_i, n_i = l_i, n_i = m_i$. A self-conjugate transformation must have

$$\gamma_i = \gamma_n = 0, \text{ if } m_i = l_i; \quad \gamma_i = \gamma_n = 0, \text{ if } n_i = l_i;$$

$$\gamma_n = \gamma_i = 0, \text{ if } n_i = m_i.$$

If $G$ is not abelian, the subgroup of its self-conjugate transformations has two or three arbitrary parameters.

Let next $n_i = 0$. If $a_i = 0$, the conditions are

$$c_i = n_i = m_i = 0, \quad b_i = c_i, \quad (l_i - m_i)a_i = (n_i - l_i)b_i,$$

$$m_i a_i = c_i n_i = l_i c_i.$$

If also $n_i = l_i$, so that $l_i = m_i$, then $\gamma_j''(j = 1, \ldots, 5)$ is symmetric in $\gamma_i$ and $\gamma_i'$ and the group is abelian. If $n_i = l_i$, and $l_i = m_i$, then $c_i = 0, b_i = 0, m_i = 0$, so that

$$z_i' = \gamma_i z_i + \gamma_i' z_i, \quad z_i' = \gamma_i z_i + (l_i - m_i)z_i + l_i z_i + l_i z_i + l_i z_i,$$

with the restrictions $a_i = 0, n_i = l_i$. The self-conjugate transformations have $\gamma_i = \gamma_i = 0$, $\gamma_i, \gamma_i'$ arbitrary. If $n_i = l_i$, and $l_i = m_i$, then $c_i = m_i = 0$, and the self-conjugate transformations have $\gamma_i = \gamma_i = \gamma_i = 0$, $\gamma_i$ and $\gamma_i$ arbitrary.

Let next $n_i = a_i = 0$. The conditions are

$$m_i b_i = c_i n_i, \quad m_i b_i = c_i n_i, \quad m_i c_i = c_i n_i, \quad (m_i - n_i)c_i = 0,$$

$$l_i b_i = b_i n_i, \quad l_i c_i = b_i n_i, \quad l_i c_i = b_i n_i, \quad (l_i - n_i)b_i = 0.$$

The transformations form an abelian group if, and only if

$$b_i = c_i, \quad l_i = m_i, \quad l_i = n_i, \quad m_i = n_i, \quad n_i c_i = c_i n_i, \quad n_i b_i = c_i n_i.$$
The form of the general transformation can be simplified by applying a transformation on $\xi_u$, $\xi_v$, and the cogredient transformation on $\eta_u$, $\eta_v$, and similarly a transformation on $\xi_w$, $\xi_x$ and one on $\eta_u$, $\eta_v$.

7. The argument of Burnside, l. c., §6, page 553, is faulty. It does not show that $\nu = \mu$, but does prove that $\nu$ is a multiple of $\mu$. In view of the work of Frobenius and that of Molien, the theorem in question is true.

The University of Chicago, May 12, 1902.

ERRORS IN LEGENDRE'S TABLES OF LINEAR DIVISORS.

BY DR. D. N. LEHMER.

Some years ago an error in Legendre's Tables of Linear Forms came to my notice. Another was found recently by members of my class, and as this error was left without correction in the later editions I determined to make a careful computation of the whole set. I was surprised to find the list of errors so long. The importance of these tables for many investigations makes it desirable that all these corrections be noted. I have also compared results with the tables in Tshebyshef's Theorie der Congruenzen, Berlin, 1889. Most of the errors in Legendre's work have been carried over uncorrected into these tables.

I. Under the form $f^2 - 29u^2$ the form $116x + 3$ should read $116x + 7$. This error was corrected in the fourth edition (1900), which is a copy of the edition of 1830.

II. Under the form $f^2 - 38u^2$ the form $152x + 129$ should read $152x + 131$. Not corrected in the fourth edition nor in Tshebyshef.

III. Under the form $f^2 - 43u^2$ the form $172x + 147$ should read $172x + 137$. Not corrected in the fourth edition nor in Tshebyshef.

IV. Under $f^2 - 51u^2$ there are two forms $204x + 13$. The second of these should read $204x + 31$. This error is in the fourth edition but not in the first (1797).

V. Under $f^2 - 61u^2$ there are so many errors that I will give the correct list: $244x + 1$, $3$, $5$, $9$, $13$, $15$, $19$, $25$, $27$, $39$, $41$, $45$, $47$, $49$, $57$, $65$, $73$, $75$, $77$, $81$, $83$, $95$, $97$, $103$, $107$, $109$, $113$, $117$, $119$, $121$, $123$, $125$, $127$, $131$, $135$, $137$, $141$, $147$, $149$, $161$, $163$, $167$, $169$, $171$, $179$, $187$, $195$, $197$, $199$, $203$, $205$, $217$, $219$, $225$, $229$, $231$, $235$, $239$, $241$, $243$. The