

lines on a cubic surface, and explain how to deduce properties of the bitangents of a plane quartic curve.

Show that there exist sets of six bitangents which touch the same conic.

4. Prove the fundamental theorems relating to the parametric expressions for the coördinates of a point on a plane curve of deficiency 0 or 1.

In the case of a plane cubic find how many lines can be drawn such that, at each point where they meet the curve, a conic can be drawn having six-point contact with the curve.

Indicate briefly how the equation of any elliptic curve can be reduced to the form

$$y^2 = 4x^3 - g_2x - g_3,$$

VIRGIL SNYDER.

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#### SHORTER NOTICES.

*Ueber die Entwicklung der Elementar-Geometrie im XIX. Jahrhundert.* Bericht der Deutschen Mathematiker-Vereinigung, erstattet von MAX SIMON. Mit 28 Figuren im Text. Leipzig, B. G. Teubner, 1906. viii + 278 pp. Price, 8 marks.

*Methodik der Elementaren Arithmetik in Verbindung mit Algebraischer Analysis.* Von MAX SIMON. Mit 9 Textfiguren. Leipzig, B. G. Teubner, 1906. vi + 108 pp. Price, 3 marks 20 pf.

AMONG those occupying chairs of mathematics in the German universities there is no one who takes greater interest in the work of the secondary teacher than Professor Simon. He is earnest in his advocacy of reform, zealous in his application of the history of mathematics to the principles of teaching, and full of that good-humored argument that makes a man acceptable as a speaker in an assembly of teachers. Therefore it comes about that Professor Simon is able to command appreciative audiences for his addresses and a goodly circle of readers for his numerous literary efforts. It is rather in his work as a speaker, however, that he is most successful. His fund of enthusiasm, his genial countenance, and his action in address, all tend to lead his hearers to consider his arguments as wholes, without attending to minor inaccuracies of language or of state-

ment. When, on the other hand, he attempts to put his ideas on paper, his larger thoughts, worthy though they may be, seem submerged in a sea of minor inaccuracies. In the case of the two books under review, the first is of such a nature as to invite even more errors than are usually found in Professor Simon's writings, and therefore, as may be inferred from what has been said, the reader will find much to vex his soul in looking over its pages.

In spite of these annoyances, however, each book has a distinct value, and each can be recommended to the teacher of mathematics, if not to the mere mathematician. The first book, "Ueber die Entwicklung der Elementar-Geometrie im XIX. Jahrhundert," is taken from the *Jahresbericht der Deutschen Mathematiker-Vereinigung*, and was originally begun in the interests of the *Encyklopädie der mathematischen Wissenschaften*. Its scope is limited, as the title suggests, in time to the nineteenth century, and in subject to the geometry of the Gymnasium or Oberrealschule. Professor Simon begins with a general survey of the bibliography of the subject, particularly that of the history, the method of teaching, and the text-books relating to geometry. It is in such lists of books and articles, imperfect though they are, that the chief value of the work will be found to lie. The author then takes up a number of special topics as follows: The theory of parallels; the circle, including quadrature, inscriptibility of polygons, and the multisection of arcs; areas, including isoperimetry; the triangle, including a study of critical points and lines; polygons; plane configurations, including similarity, centers of gravity, and transversals; general space relations, in particular certain questions of stereometry, volumes and elementary surfaces, and spherics; special space relations, including the tetrahedron, polyhedron, and the Euler theorem; and trigonometry, plane and spherical.

Under each of these topics a brief historical survey is given, with a bibliography, and this is followed by a summary of the most important attempts at treating the subject in hand. Thus under the quadrature of the circle the historical survey is followed by a list (rather imperfect) of circle-squarers, a statement of approximate constructions, a few of the efforts at representing  $\pi$  numerically, and the history of a few such attempts as that of Hippocrates. In the same way the trisection problem, the 17-gon, the critical points of the triangle, and Euler's

problem (with no reference to Hadamard's proof) are discussed. In no case is the treatment as thorough as in Klein's *Vorträge*, but it is always readable, and the bibliography opens up to the student a good part of the literature of the subject.

The "Methodik der elementaren Arithmetik" is a set of lectures delivered by Professor Simon in the summer semester of 1904. As such, they would naturally be expected to be more popular and less critical in details than the work already mentioned. They are simple in style, and were doubtless intended for teachers who felt the need of having their attention called to the modern treatment of certain fundamental ideas of mathematics. Hence we find a popular treatment of the idea of integral number, the operations with number and the fundamental laws involved, fractions, series, powers and roots, equations of the first three degrees, logarithms, and complex numbers. It is such a course of lectures as might profitably be attended by the high school teacher in this country, and the book should find a considerable circle of readers here as well as in Germany.

While the books have this value, however, it must be said that the array of inaccuracies which confronts the reader seems quite inexcusable. The number of the errata at the end of the *Entwicklung*, upwards of 135, might easily be doubled from a single reading. Moreover there are other and more serious points of criticism. Professor Simon's dismissal of Dr. H. Schotten's "Inhalt und Methode" by the simple word "schwächer" is unwarranted, for the book contains a good deal that is of value to the teacher of geometry. His statement that Rupert's little book on famous propositions of geometry seems to be merely an extract from Ball cannot be justified; if the book were to be attacked it should be on other grounds. Moreover Professor Simon's knowledge of the relative values of books, monographs, and proofs of a non-German character is limited, as the merest glance at his list of text-books will show. An example of this is seen in his inclusion of an edition of Sacrobosco's *Sphæra* in a list of works on modern spherics. His statement (Methodik, p. 16) that in Sargon's time the position system was known is liable to serious misconstruction. His statement that the French call the multiplication table the "table de Pythagore" will also be misunderstood, for the name was a common one in the works of the mediæval writers. That Gemma Frisius said "unitas ipsa non est numerus," in 1576

(Methodik, page 8), is impossible, for he was dead then. Moreover it was not at all a new idea when he published his arithmetic in 1540, for it had been generally repeated by writers from the time of Boethius down. His statement that the study of the Pascal triangle began with Stifel is also incorrect, for it was apparently known long before Stifel's works were written, appearing for example in the engraved title page of "Eyn Neue Vnnd wolgegründte vnderweysung aller Kauffmanss Rechnung" published by Apianus at Ingolstadt in 1527.

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*Elements of Descriptive Geometry.* By O. E. RANDALL. Boston, Ginn & Company, 1905. iv + 209 pp.

*Elements of Descriptive Geometry.* By CHARLES E. FERRIS. New York, American Book Company, 1905. vii + 127 pp.

THESE books meet the demand for texts in which the objects are shown in the figures in the third quadrant. They follow the method of Warren and Church in presenting first the analysis and then the construction of a given problem. The figures are not in a separate volume or collection of plates, but in the text where most convenient for the reader.

The authors present the fundamental ideas with care, and Dr. Randall supplements the figures for the first fifty pages with "pictorial drawings" showing the projection planes and the object as they actually appear to the eye of the observer.

It will be noticed that the chapters on "Lines and surfaces" retain the definitions and principles used by the older American writers. If it is necessary to set up for engineering students a complete theory of curves and surfaces without analysis, it is desirable that in doing it naïve expressions that may confuse students of mathematics should be avoided.\* For example, in Dr. Randall's book, § 203, we read \* \* \* "The portion of the line generated by the point while moving from one position to its consecutive position is called an *elementary line* and while in theory it may be regarded as having length, practically speaking it has none."

The error of classifying a warped surface as a surface of single curvature is implied by Dr. Randall in § 221, where we find :

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\* Compare Rohn, Lehrbuch der darstellenden Geometrie, Leipzig, 1906, p. 219.