approximate construction for $\pi$.  

angles. This approximation is then made closer by using the values of $f$ at points where $AB$ cuts the curve $f(x, y) = \text{const.}$ If the second approximation is not close enough, the process is repeated.

23. Herr Wagenmann correlates successive steps in the theory of evolution with series $-\infty, \cdots, -2, -1, 0, 1, 2, \cdots, \infty$ along three coordinate axes developing successively the ideas of motion, mass, the nebular hypothesis and evolution of living organisms and of civilization. He finds that his method leads to a monistic philosophy—in fact to a pan-monism.

Göttingen,  
November, 1906.

A NEW APPROXIMATE CONSTRUCTION FOR $\pi$.

By Mr. George Peirce.

Given a circle with radius $r$ and center at $O$; to find an approximate construction for $\pi r$.

Draw the diameter $AOB$ and the tangent $BC$ at right angles to it. Describe the arc $ODC$ with radius $r$ and center at $B$.

Draw the line $AC$ cutting the arcs $ODC$ and $AB$ at $D$ and $J$; also draw the line $BDE$ through $B$ and $D$ cutting the given circle at $E$. Then $AD + 3DE = \pi r$ approximately.
1907]. APPROXIMATE CONSTRUCTION FOR π. 167

Proof:

\[ AC = \sqrt{AB^2 + BC^2} = r\sqrt{5} \]

\[ AD = \frac{AO \cdot AH}{AC} = \frac{r \cdot 3r}{r\sqrt{5}} = \frac{3}{5}\sqrt{5}r, \]

\[ JC = \frac{BC^2}{AC} = \frac{r^2}{r\sqrt{5}} = \frac{1}{5}\sqrt{5}r \]

\[ DJ = AC - AD - JC = \frac{1}{5}\sqrt{5}r, \]

\[ DE = \frac{AD \cdot DJ}{BD} = \frac{\frac{3}{5}\sqrt{5}r \cdot \frac{1}{5}\sqrt{5}r}{r} = \frac{3}{5}r, \]

\[ AD + 3DE = \frac{3}{5}\sqrt{5}r + 3\left(\frac{3}{5}r\right) = 3.141641r. \]

By making use of the fact that in the triangle \( ABE \)

\[ AE = \sqrt{(AB^2 - BE^2)} = \sqrt{(2r)^2 - \left(\frac{3}{5}r\right)^2} = \frac{2}{5}r = 2DE, \]

we can obtain a single line of the same length as \( AD + 3DE \).

We can therefore draw the arc \( EG \) with radius \( DE \) and center at \( D \) and the arc \( EF \) with radius \( AE \) and center at \( A \). Then \( AD + 3DE = AD + AE + DE = AD + FA + DJ = FG \).

There are many other approximate constructions for \( \pi r \). A summary of those that have been worked out according to the method of geometrography is given below. \( A, B, C \) and \( D \) are to be found in the BULLETIN for January, 1902, page 137; \( E \) is in Cantor’s Geschichte der Mathematik, volume 3, page 23; \( F \) is the construction given above.

<table>
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<th>Author</th>
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\( Δ \) is the difference between the mechanically exact construction and \( πr \). \( S \) stands for simplicity and \( E \) for exactitude.

For the technical meanings of these two words see the article in the BULLETIN for January, 1902. The lower these numbers are, the better the construction.