

avoiding oblique axes. Later a justification for this procedure appears as the result of successive multiplication of the vector 1 by  $i$ . The chapter ends with a trigonometric treatment of the roots of unity.

The remaining three chapters are on combinations and permutations, the binomial formula, equations of the first degree and determinants. Everywhere we recognize the resolute endeavor to present the subject with perfect sincerity. The author says in the preface: "J' ai horreur d'un enseignement qui n'est pas toujours sincère: le respect de la vérité est la première leçon morale, sinon la seule, qu'on puisse tirer de l'étude des sciences."

F. CAJORI.

*Leçons d'Algèbre et d'Analyse.* Par JULES TANNERY. Tome Second. Paris, Gauthier-Villars, 1906. 638 pp.

THIS volume, which is the second of a comprehensive treatise on this field of algebra and analysis, like the other volumes, is prepared for the use of special students of mathematics of the Sorbonne, Paris. The chapter headings are as follows: series, functions of a real variable, series of functions, applications in the study of a function in the separation and calculation of the roots of an equation, algebraic equations, differential notation and plane curves, and notions of integral calculus.

The chapter on series deduces, with a simplicity and clearness that are ample for beginners of collegiate grade, the fundamental notions of the subject, establishes the ordinary tests for convergence and divergence, examines for convergence many series of frequent occurrence, and closes with seven pages of interesting exercises on convergence, divergence, and equivalence of series. All this is done in the compass of 53 pages and that, too, without material omission of essentials.

The meanings of variable, of function, of the phrases "appertaining to an interval" and "lying within an interval," and of bounds are given and exemplified in the next chapter on functions of a real variable. Common geometric notions are first given, then shown to be too indefinite for the purposes of analysis; then the analytic definitions are given for the following: curve, continuity, functionality, and increasing, decreasing, and discontinuous functions. The meaning and domains of validity of inverse, logarithmic, circular, and exponential functions are pointed out. A development of the properties of

hyperbolic functions completes the theoretical part of the chapter. The principles deduced are reviewed and applied by a four-page list of well-selected exercises.

The third chapter renders precise and trustworthy the terminology and basic notions involved in deriving functions of real variables, and gives general methods of deriving, both totally and partially, functions of real variables with real, and with imaginary coefficients. Rolle's theorem and the so-called theorem of finite differences are deduced and four pages of exercises are added. The functions here treated being capable of graphic representation, the author satisfies himself with the degree of rigor which such functions require, and postpones for later consideration the restrictions necessary for absolute analytical rigor. At appropriate places the student is cautioned as to the inadequacy of these first treatments. He is thus led to feel that while he is always getting the truth, and as much of it as he can assimilate, yet it is not the *whole* truth. This procedure cultivates in the student open-mindedness and the feeling that there is still more to learn.

Series whose terms are functions of a variable, series in integral powers of a single variable—power series—first with real, then with imaginary coefficients, and the Taylor and MacLaurin developments are carefully treated and extensively applied to both algebraic and transcendental functions in the next chapter. Several illusory forms are given special attention and their true values are defined.

Then follows a study of variations of functions, of roots, multiple roots, asymptotes, convexity, concavity, points of inflexion, and separation of roots. Rolle's theorem is again taken up, and methods of approximating roots of functions are taught and given extensive application.

The relations of coefficients and of roots of algebraic equations, symmetric functions, Waring's method, methods of elimination and transformation, reciprocal, binomial, cubic, and quartic equations, together with other topics of the theory of algebraic equations, are given full and yet concise consideration. The differential and determinant notations applied to the study of plane curves, with a clearly logical and lucid development of the integral calculus, running through 154 pages, constitute the next chapter, which completes the volume.

From this synopsis it is plain that the French order of procedure—for M. Tannery's order is customary in France—

through the several branches of mathematical analysis differs materially from the order of American texts. Of course, it does not follow that because another nation prefers a certain plan it is the best thing for us to adopt the plan. It must be admitted, however, that the loosely articulated subject matter as given in our so-called college algebra, which closes with a rudimentary and isolated treatment of the theory of equations, then trigonometry, then analytics, then calculus, leaves much to be desired. Many desirable things might be suggested by the close study of such masterly treatments as this of M. Tannery by would-be authors of college algebras and of other mathematical books for use in our schools. Both in point of gradation of inherent difficulties and in point of simplicity or treatment without loss of rigor the French text-book writers can teach us much. The logical order and mode of this volume are admirable.

Again, the plan by which M. Tannery gradually inducts the learner into the logical niceties of the subject, by giving him from time to time enough to satisfy present needs and as much as his stage of mathematical maturity and advance justifies, then reverting to the matter later, when experience has ripened sufficiently to enable the novice to interpret what he is doing, is a distinct advance pedagogically upon the common practice of American college professors. The latter are too prone to seek fulness of detail the first time the subject is taken up. The reviewer would invite their attention to M. Tannery's procedure, for these *Leçons* were evidently prepared to inform and to educate rather than merely to present a beautifully perfect mathematical exhibit.

Only half a dozen typographical errors, and these of no particular consequence, have been discovered in the entire volume. Typography and general mechanical excellence are sufficiently guaranteed by the imprint of Gauthier-Villars.

G. W. MYERS.

*Mathématiques. Principes et Formules de Trigonométrie Rectiligne et Sphérique.* Par J. PIONCHON, Professeur à la Faculté des Sciences, Directeur de l'Institut Electrotechnique de l'Université de Grenoble. Paris, Gauthier-Villars, et Grenoble, A. Gratier et J. Rey, 1904. 5 francs.

THIS is one of the 70 odd volumes now appearing from the press of Gratier et J. Rey, of Grenoble, under the editorial