

culties in the demonstrations to which these quantities give rise are surmounted by a device which again is suggested by measurement and which exhibits all the rigor that the student is likely to be able to appreciate.

It will hardly be questioned that this text will appeal more strongly to the students' interest than Euclid, nor that the material is better selected with reference to the students' capacity to receive it, nor that the student can, by the expenditure of a given amount of energy, obtain a greater amount of mathematical information from this text than from Euclid's Elements. It still remains in doubt, however, whether the student will obtain the same thorough training in rigorous, careful reasoning in this course as under the present discipline.

C. H. SISAM.

*The Foundations of Mathematics.* A Contribution to the Philosophy of Geometry. By Dr. PAUL CARUS. Chicago, The Open Court Publishing Co., 1908. 141 pp.

THIS book is, mathematically speaking, a more or less popular treatise, which would appear to have for its primary object an effort to show that geometry can be obtained a priori, by abstraction, from the notion of motility, and can be constructed from this alone by making use of the principles of reasoning, *all axioms being unnecessary.*

The book opens with a historical sketch, which is fairly accurate, mentioning particularly the work of Euclid, Gauss, Riemann, Lobachevsky, Bolyai, Cayley, Klein, and Grassmann. The author then introduces chapters on "The philosophical basis of mathematics" and "mathematics and metageometry" in which his philosophical theories are presented. Briefly expressed, his doctrine seems to be about as follows: "Space is the possibility of motion, and by ideally moving about in all possible directions, the number of which is inexhaustible, we construct our notion of pure space. If we speak of space we mean this construction of our mobility. It is an a priori construction and is as unique as logic or arithmetic. There is but one space, and all spaces are but portions of this construction." Mathematical space is a priori, in the Kantian sense, not however ready made in the mind, but the product of much toil and careful thought. Mathematical space is an ideal construction, hence all mathematical problems must be settled by a priori operations of pure thought, and can not be decided by external

experiment or by reference to a posteriori information. Space being obtained by abstraction, is unique, and has definite properties, and requires no *axioms* for its development. The theory of parallels is only a side issue of the implications of the straight line. The author leads the reader to expect the conclusion that Euclid alone is valid, yet he says later (page 121), "The result of our argument is quite conservative. It reestablishes the apriority of mathematical space, yet in doing so it justifies the method of metaphysicians in their constructions of the several non-euclidean systems."

There is much vagueness and apparent contradiction in the book. The abstraction process, except in so far as it is purely intuitional, would seem, if definite at all, to be nothing more than an arbitrary process, and hence equivalent to a set of axioms. The author is not concerned with any question of betweenness, or of continuity, except as involved in notions of homogeneity, evenness, his interest being almost entirely in the parallel axiom and its implications.

The book concludes with an epilogue in which the analogy between mathematics and religion is discussed, although the precise analogy is not quite clear.

F. W. OWENS.

*Mechanics.* By JOHN COX. Cambridge University Press (Cambridge Physical Series), 1904. Demy 8vo. xiv + 332 pp.

THIS book ought to have a far reaching influence on the teaching of elementary mechanics. It contains really good illustrative examples, concrete, practical, and instructive, and at the same time, gives clear and accurate statements of the fundamental principles. It is not overloaded with theory more general than ordinary applications require. Further, principles are expressed in words rather than by formulas. In simple examples it is clumsy to use a general formula, in complicated examples verbal expression often clears the view, in all examples the mere substitution of numerical values in a formula is poor practice.

Two paragraphs from the author's preface are worth quoting. "Some years ago I stumbled on the first German edition of Professor Mach's *Die Mechanik in ihrer Entwicklung*. . . . Since then my teaching has been based more and more on the lines laid down by Mach, and as I have found it impossible to