

of the Möbius net. The three numbers appearing in the expression for this vector and by which the point is fixed are called the anharmonic coordinates of the point. In chapter III, the equation of the straight line is developed from a condition on the coefficients of three coinital vectors. Among the subjects treated in the other chapters are the general equation of the second degree, special conics, tangential equations, the anharmonic ratio, the involution, circles, and foci.

In regard to some details in the book, the reviewer would suggest omitting the words "of intersection" from line 7, section 8°, on page 14. Also it would seem better to use the parameters t and v homogeneously throughout section 9°, pages 14 and 15. In equation (16), page 17, read $\cos C(p_1q_2 + p_2q_1)$ instead of $\cos C(p_1q_1 + p_2q_1)$. The next form of this same equation displays without warning a change of notation that at first glance is rather puzzling. Half a line would state the change clearly. In line 5, page 20, read P_1' and P_2' for P_1 and P_2 . In the line following equation (1), page 21, read Σ^2x_2 for Σx_2 . In the equation near the bottom of page 27, read $2(f\varphi_{x'} + g\varphi_{y'} + h\varphi_{z'})t$ for $2(f\varphi_{x'} + g\varphi_{y'} + h\varphi_{z'})$. In line 9, page 51, read X for IX. In line 10, page 54, read "the" for "some." At the bottom of page 63, read $p|pq_2r_3|$ and $p|pq_4r_3|$ for $r|pq_3r_2|$ and $r|pq_3r_4|$, the values of t and t' respectively. In line 2, section 5°, page 65, read D' for D . The value given for $C'D'$, page 67, is the reciprocal of the correct value. Likewise for the value of $B'C'$, and in addition read $|x_2y_3|$ for $|x_2y_2|$. The ditto marks on page 87 neglect the factor $a^2b^2c^2$. In the value for y'/z' , page 88, read $|xy_1z_2|$ for $|xy_2z_2|$. In line 3 from the bottom of page 93, read c^2 for c^3 .

These items suggest that the book is a little loosely put together in some respects; but it contains nevertheless much valuable material.

J. V. McKELVEY.

Algebraische Kurven. Zweiter Teil: Theorie und Kurven dritter und vierter Ordnung. By EUGEN BEUTEL. Sammlung Göschen No. 436. Leipzig, 1911. 16mo. 135 pp. Price, 80 Pf.

In a thin book of pocket size this treatise gives a large number of most precise definitions and theorems, fifty-seven well-executed cuts, and a variety of carefully worked out nu-

merical examples for illustration. Fifty pages are given to polars, the Hessian, duality, Plücker's formulas, and higher singularities. Curves of the third order fill thirty pages; curves of the fourth order, fifty; and there is an excellent index with nearly 150 references. Of course much is statement without proof, as in Pascal's Repertorium and the Encyclopädie; but a very considerable body of concise proof is included. The cuts are a specially admirable feature; many teachers who use lanterns in lectures will find them more available than those in Loria's collection.

Of the great mass of known theorems, for the most part only those are chosen which have direct bearing on the visible representation and the classification of curves; but this restriction permits relative fulness within limits. Concerning the related theory of invariants, elliptic and abelian functions, and covariant systems of curves, there is almost nothing here. But what is given is just what the beginner requires.

Extracts may be given to show that the author has his own point of view. "Zugleich ist ersichtlich, dass bei Anwendung von Linienkoordinaten die Aufgabe: die Umhüllungskurve einer nach einem bestimmten Gesetz sich bewegenden Geraden zu finden, rein algebraischer Natur ist; für Punktkoordinaten ist dies eine Aufgabe der Differentialrechnung" (page 31, § 3). Unless something is premised concerning the nature of the Gesetz and the terms in which it is expressed, this is a rash assertion. The statement intended was perhaps that a limit process is required in deducing the line equation of a curve from its point equation, or vice versa. On this problem the author gives a lucid discussion and (pages 38–41) very useful hints and examples.

As to ordinary and singular points and tangents (page 44) we find it stated that on point loci inflexional points and double tangents are ordinary features, while they are singularities on line loci; and dually for cusp tangent and double point. In spite of a plausible reason for this choice of words, it seems to the reviewer that the usual mode is better, namely to speak of inflexional *tangents*, with double tangents, as singularities on line loci, etc. For the one kind of *tangent* is, no less than the other, a part of the *projective* entity that we mean by point curve, and both alike are explicitly referred to in the line equation; we do not see how the substitution of the point of contact in place of its tangent can fail to confuse the student.

With these meticulous criticisms we may join a third. On page 67 we read: "Zwei Kurven derselben Klasse sind aber nicht immer ineinander projizierbar (vgl. z. B. die Dreiecks- und Viereckskurve dritter Ordnung S. 70). Die Einteilung nach der Klasse ist also keine projektive." A non sequitur is an agreeable rarity, and this has evidently, from the context, slipped in through some oversight.

In laying down this multum in parvo, we must commend the sections on quadratic transformation and the cuts exhibiting conics and their various related rational quartics.

H. S. WHITE.

Lehrbuch der Mathematik für Studierende der Naturwissenschaften und der Technik. Einführung in die Differential- und Integralrechnung und in die analytische Geometrie. Von Dr. GEORG SCHEFFERS. Zweite verbesserte Auflage. Leipzig, Veit, 1911. 8vo. vi+732 pp.

IMAGINE a course of some 150 lectures on algebra, trigonometry, analytic geometry, and calculus given by a sound mathematician and an excellent teacher, not lacking in the sense of humor. Imagine the audience to be students of general science or engineering who have taken the usual secondary school courses in mathematics, "jedoch manches davon wieder vergessen, vielleicht auch manches davon nicht ganz verstanden haben." (From the preface.) Imagine these lectures together with all side remarks, illustrations and black-board drawings and sketches taken down word for word by a good stenographer, whose notes are transcribed and published in a large octavo volume by a first rank Leipzig firm. Imagine all this and from one point of view the reader will have a good idea of the book under review.

The word "function" dominates the plan of the work. If we call our usual division of college mathematics into algebra, trigonometry, analytic geometry, and calculus a horizontal division, we might call Scheffers' division a vertical one. Beginning with the notion of a function, he takes up one after the other, linear, quadratic, rational integral, rational, logarithmic, exponential, and trigonometric functions. An outline of his chapter on the quadratic function will give an idea of his method of treatment. The graph of the function is discussed in great detail, beginning with the simple case x^2 and then taking up more complicated cases with numerical