

The Publikum then asks for the cube root of 45,118,016, and the Rechenkünstler at once states that it is 356. When asked for the cube root of a perfect cube of seven figures he calls for the last three figures only, and upon being told that they are  $\dots 313$  he at once says that the cube root must be 217, although he is ignorant of the other figures.

The Publikum then sets the problem to find the seventh root of a twelve-figure number, and is asked to state the last figure (in this case  $\dots 7$ ) and then to give all the figures in any order it chooses (in this case given as 887621111107, although the whole number was really 271,818,611,107). The answer is at once given as 43.

Among some of the more difficult problems worked out mentally, the method being stated in the text, are the following:

The fifth root of 11,576,155,017,345,132,257 is 6497. The eleventh root of a number of fifteen figures (952,809,757,913,927) is given as 23, the computer being told the figures in any order whatever (in this case, 012235577789999).

The thirty-first root of a perfect power, the number having thirty-five places, is given as 13, the computer not being told even a single figure. The number is 34,059,943,367,449,284,484,947,168,626,829,637.

The second half of the book is devoted to the Easter problem, the famous Elberfeld horses, the multiplication methods of Ferrol (which are shown to be applications of processes known for many centuries), the relation of the properties of nine to the Fermat problem, and the further proposition of Fermat with respect to  $a^{p-1}$ .

DAVID EUGENE SMITH.

*Konforme Abbildung einfach-zusammenhängender Bereiche.*

Von E. STUDY. Zweites Heft, herausgegeben unter Mitwirkung von W. BLASCHKE. Leipzig, B. G. Teubner, 1913. iv + 142 pp.

THIS is the second volume\* of a series of lectures on geometric topics by Study and deals with the conformal transformation of singly connected domains, a subject which, after a long period of stagnation, has in recent years received the attention of a number of investigators. In the center of these researches stands the possibility of the conformal mapping of a given

---

\* See review of vol. 1, in this BULLETIN, vol. 19, pp. 15-18.

singly connected domain upon a circle, which was definitely proved almost simultaneously by Poincaré\* and Koebe† in 1907. After the definition of a singly connected domain and its different types we find in §§ 1–4 chiefly a reproduction of Koebe's investigations leading to the principal theorem. This is followed by a detailed discussion of the intricate problems which arise in connection with the mapping of the points on the boundary of the domain (§§ 5–8). The reader of this portion of the book is expected to be equipped with a fair knowledge of function theory and is repeatedly referred to Osgood's "Lehrbuch," whose second edition appeared while the book under review was in press. The results obtained by Koebe are presented by Blaschke. In case the boundaries of the domain are Jordan curves there are still some difficulties in the way, and to overcome these, the authors, on page 65, introduce the hypothesis that the map of a radius of the circle  $K$ , which is conformally mapped upon the domain  $B$ , is an analytic single curve in  $B$  terminating in a "Kernmenge"  $\Omega(\zeta)$ .

They hope, however, that the problems concerning the boundary points which only a short time ago were considered as almost unapproachable will soon find their complete solution, and also that a simpler method for these theories may be established.

In the second part, §§ 9–16, the problem of conformal mapping of a circle upon a convex domain is solved and the well known formula found by Christoffel and H. A. Schwarz is generalized. The treatment is, as may be expected, very rigorous and includes the "two-side" and "one-side" as improper polygons.

Instead of starting with the formula

$$w = C \int_{z_0}^{z^z} (z - z_1)^{\mu_1} (z - z_2)^{\mu_2} \dots (z - z_n)^{\mu_n} dz$$

as given in advance, we should prefer its derivation by the beautiful method in Darboux's *Théorie générale des Surfaces*.‡ Here the converse problem is solved, to map a given polygon upon a circle, or upon the upper half-plane.

\* *Acta Mathematica*, vol. 31, pp. 1–63, printed on March 19, 1907.

† *Gött. Nachr.*, May 11, 1907, pp. 175–210; Nov. 23, 1907, pp. 633–669.

‡ Vol. 1, pp. 170–192.

Making use of the angle  $\Theta$  which the "support" (Stütze) of a polygon makes with a fixed directed line and which may be defined as a function (Stützwinkelfunktion) of the parameter  $\vartheta$  in the parametric representation

$$\zeta = e^{i\vartheta}$$

and the unit circle and Stieltjes' integrals, §§ 11-12, the mapping process is extended to simple convex domains in an original manner (§§ 13-14).

Theorem 7, page 66, which is based upon the hypothesis made on the previous page is rigorously proved in § 15 by means of Koebe's famous "Verzerrungssatz" (theorem of distortion). In conclusion references to some applications of Koebe's theorem and continuity method are given. We merely mention the interesting theorem: Every singly sheeted domain of the plane with  $n$ -fold connectivity can always be mapped conformally upon another singly sheeted domain whose boundary consists of  $n$  rectilinear cuts of given directions.

Many a reader would have probably greatly appreciated a fuller treatment of Koebe's methods and results indicated at the end of this section, and would certainly welcome another volume on these more advanced subjects together with the results quite recently obtained by Plemelj, Carathéodory, Osgood, and others.

As a most valuable feature of the book I mention the interesting examples worked out in the last section.

ARNOLD EMCH.

---

## NOTES

THE April number (volume 36, number 2) of the *American Journal of Mathematics* contains the following papers: "Iterated limits in general analysis," by R. E. ROOT; "Simply transitive primitive groups whose maximal subgroup contains a transitive constituent of order  $p^2$ , or  $pq$ , or a transitive constituent of degree 5," by Miss E. R. BENNETT; "An extension of Green's theorem," by Miss I. BARNEY; "On the asymptotic solutions of linear differential equations," by C. E. LOVE; "Restricted systems of equations," by A. B. COBLE; "The canonical types of nets of modular conics," by A. H. WILSON; "On long waves," by J. H. M. WEDDERBURN.