For \( l = k \) it follows that
\[
 a_{k-j, j} + ib_{k-j, j} = (i)^j(a_{k0} + ib_{k0}) \quad (j = 1, \ldots, k).
\]
Then
\[
 W(z) = \sum_{k=0}^{\infty} (a_{k0} + ib_{k0})(x + iy)^k
 = \sum_{k=0}^{\infty} C_k z^k, \text{ where } (C_k = a_{k0} + ib_{k0}).
\]
This completes the theorem.

I am indebted to Professor E. J. Townsend for suggesting the problem.

Ohio University, Athens, Ohio.

CONCERNING THE COMPLEMENT OF A COUNTABLE INFINITY OF POINT SETS OF A CERTAIN TYPE.

BY DR. J. R. KLINE.

(Read before the American Mathematical Society, December 27, 1916.)

In his “Grundzüge der Mengenlehre,” Hausdorff proved that if \( E \) denotes a euclidean space of two or more dimensions while \( R \) is a countable set of points belonging to \( E \), then \( E - R \) is a connected point set.\footnote{Cf. F. Hausdorff, “Grundzüge der Mengenlehre,” Leipzig, Veit, 1914, p. 333.} It is the object of the present paper to prove a theorem, which contains Hausdorff’s theorem as a special case. Hausdorff’s method of proof does not apply for the proof of the more general theorem. While the proof is carried out for the case of two dimensions, it is evident that a similar proof would apply to any higher number of dimensions.

THEOREM. If \( M \) is a domain\footnote{\( M \) a domain is a connected set of points \( M \), such that if \( P \) is a point of \( M \), then there exists a region containing \( P \) and lying in \( M \).} and \( G_1, G_2, G_3, \ldots \) is a countable infinity of nowhere dense\footnote{\( A \) set of points is said to be nowhere dense, if in every region \( R_1 \) there exists a region \( R_2 \) entirely free of points of the set. A set is said to be closed if it contains all its limit points.} closed point sets, no one of

* A set of points is said to be connected if, however it be divided into two mutually exclusive proper subsets, one of them contains a limit point of the other.

\footnotetext{\( M \) a domain is a connected set of points \( M \), such that if \( P \) is a point of \( M \), then there exists a region containing \( P \) and lying in \( M \).}

\footnotetext{\( A \) set of points is said to be nowhere dense, if in every region \( R_1 \) there exists a region \( R_2 \) entirely free of points of the set. A set is said to be closed if it contains all its limit points.}
which disconnects any domain, then $M - (G_1 + G_2 + G_3 + \cdots)$ is connected.

Proof.*—Let $A$ and $B$ denote any two distinct points of $M - (G_1 + G_2 + \cdots)$.† The point set $M - G_1$ is a domain. For each point $P$ of $M - G_1$, let $K_{1P}$ denote a region, of subscript greater than or equal to 1, which belongs to the fundamental sequence‡ and is such that $K'_{1P}$§ is a subset of $M - G_1$. Call the set of all such regions $S_1$. There exists a simple chain|| $R_{11}, R_{12}, R_{13}, \ldots, R_{1n_1}$ from $A$ to $B$ every link of which is a region of the set $S_1$. Call this chain $C_1$. As two regions which have a point in common, also have in common a region containing that point, and as $G_2$ is a nowhere dense closed point set, it follows that $R_{1i}$ and $R_{1i+1}$ $(i = 1, 2, \ldots, n_1 - 1)$ have in common a point $P_{1i}$, not belonging to $G_2$. Call $A$, $P_{10}$ and $B$, $P_{1n_1}$. For each point $P$ of $R_{1i}$ $(i = 1, 2, 3, \ldots, n_1)$ which does not belong to $G_2$, let $K_{2P}$ denote a region of the set $K_2, K_3, \ldots$, which is such that $K'_{2P}$ lies in $R_{1i}$ and contains no point of $G_2$. Call the set of all such regions $S_2$. It is now possible to construct a simple chain $C_2$, which satisfies all the requirements of Professor Moore's $C_2$¶ and has the additional property that every link of $C_2$ belongs to $S_2$ and therefore contains no points of $G_1 + G_2$. Continue this process. We obtain an infinite sequence of chains $C_1, C_2, C_3, \ldots$ which satisfy all the requirements of Moore's sequence of chains $C_1, C_2, C_3, \ldots$ and have the additional property that no link of the chain $C_n$ has a point in common with the set $G_1 + G_2$

*Our theorem is proved on the basis of the system of axioms $\Sigma_1$ proposed by R. L. Moore in his paper, "On the foundations of plane analysis situs," Transactions Amer. Math. Soc., vol. 17 (1916), pp. 131–164.

† That $M - (G_1 + G_2 + G_3 + \cdots)$ contains infinitely many points follows at once from one of the theorems of Baire, Annali di Mat. (3), vol. 3, p. 65.

‡ Select once for all a definite sequence, $K_1, K_2, \ldots$ satisfying the conditions of Axiom 1 of $\Sigma_1$. This definite sequence will be called the fundamental sequence and its regions will be termed fundamental regions.

§ The boundary of a point set $M$ is the set of all limit points of $M$, which do not belong to $M$. If $R$ is a region, $R'$ denotes the point set composed of $R$ plus its boundary.

|| If $A$ and $B$ are distinct points, then a simple chain from $A$ to $B$ is defined by R. L. Moore as a finite sequence of regions $R_1, R_2, R_3, \ldots, R_n$ such that (1) $R_1$ contains $A$ if and only if $i = 1$, (2) $R_i$ contains $B$ if and only if $i = n$, (3) if $1 \leq i \leq n$ and $1 \leq j \leq n$, while $i < j$, then $R_i$ has a point in common with $R_j$ if and only if $j = i + 1$. The region $R_k$ $(1 \leq k \leq n)$ is said to be the $k$th link of the chain. See R. L. Moore, loc. cit., p. 134. For a proof of the existence of such a chain, see R. L. Moore, loc. cit., Theorem 10, p. 135.

¶ Cf. R. L. Moore, loc. cit., p. 137.
AN ANALOGUE TO PASCAL’S THEOREM. [Mar.,

Let \( C_n \) denote the point set which is the sum of all the links of the chain \( C_n \), while \( C \) denotes the set of all points that the sets \( C_1, C_2, C_3, \ldots \) have in common. The point set \( C \) is a simple continuous arc* from \( A \) to \( B \), lying entirely in the set \( M - (G_1 + G_2 + \cdots) \).†

It follows that \( M - (G_1 + G_2 + \cdots) \) is connected.

UNIVERSITY OF PENNSYLVANIA,
PHILADELPHIA, PA.

AN ANALOGUE TO PASCAL’S THEOREM.

BY DR. A. L. MILLER.

A DECAGON is said to be doubly inscribed in a cubic if every odd side of the decagon cuts three even sides on the cubic and every even side cuts three odd sides on the cubic.

That there exist decagons doubly inscribed in a cubic can be seen as follows. Let the decagon \( D \) have for sides \( s_1, s_2, s_3, \ldots, s_{10} \) and let the cubic be \( C_3 \). Let

\[
\begin{align*}
& s_1 \text{ meet } s_{10}, s_2, s_4 \text{ on } C_3, \\
& s_3 \text{ meet } s_2, s_4, s_6 \text{ on } C_3, \\
& s_5 \text{ meet } s_4, s_5, s_8 \text{ on } C_3, \\
& s_7 \text{ meet } s_6, s_8, s_{10} \text{ on } C_3,
\end{align*}
\]

while \( s_9 \) is the line joining the third intersection of \( s_8 \) with \( C_3 \) with the third intersection of \( s_{10} \) with \( C_3 \). Then, by Cayley’s‡ theorem, \( s_9 \) also cuts \( s_2 \) on \( C_3 \).

By a repetition of this last theorem we obtain the following theorem analogous to Pascal’s theorem:

If a decagon be doubly inscribed in a cubic the remaining ten intersections of the odd sides with the even ones lie on a conic.

UNIVERSITY OF MICHIGAN.

* If \( A \) and \( B \) are distinct points, a simple continuous arc from \( A \) to \( B \) is defined by Lennes as a bounded, closed, connected set of points containing \( A \) and \( B \), but containing no proper connected subset containing both \( A \) and \( B \). See N. J. Lennes, “Curves in non-metrical analysis situs with an application in the calculus of variations,” American Journal of Mathematics, vol. 33 (1911) and this BULLETIN, vol. 12 (1906), p. 284.

† For a proof of this statement, see the proof of Theorem 15 of Moore’s paper, loc. cit., pp. 136-9.

‡ Cayley: Cambridge and Dublin Mathematical Journal, vol. 3.