

The rest of the proof of the theorem is almost immediate. In consequence of the continuity of $\sigma(t)$ there corresponds to every value of σ between 0 and $\lambda = \sigma(1)$ at least one value of t , and so at least one point of the curve. There can not be more than one point for a single value of σ , because the values of σ corresponding to any two distinct points differ by at least $\psi(\delta)$, if δ is the length of the chord joining the points. If we set $x = F(\sigma)$, $y = \Phi(\sigma)$, these are single-valued functions of σ , and are identically equal to $f(t)$ and $\varphi(t)$ respectively, by their very definition. They are continuous, because

$$\Delta\sigma \geq \psi(\sqrt{\Delta x^2 + \Delta y^2}), \quad \Delta x \leq \sqrt{\Delta x^2 + \Delta y^2} \leq \omega(\Delta\sigma) + \Delta\sigma,$$

and similarly for Δy . Finally they are not constant together in any interval, by the same argument as in the case of a rectifiable curve.

The extension of the work to curves in three or more dimensions requires only the writing down of a correspondingly larger number of symbols.

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JOHN WALLIS AS A CRYPTOGRAPHER.

BY PROFESSOR DAVID EUGENE SMITH.

It is not probable that many bibliophiles in the domain of mathematics, seeing upon their shelves the sumptuous tall copies of the *Opera Mathematica** of John Wallis, and consulting their noteworthy historical chapters, the first serious effort in the history of mathematics in England, ever consider that the author was one of the world's greatest decipherers of cryptic writing. To be sure his biographies give us the information that he was interested in cryptography, but the extent of this interest, the sixty years devoted to the subject, the services rendered to the State, the rewards and disappointments that came to him as a result—of all this the biographies tell us practically nothing. It is partly because of this fact,

* *Oxonisæ*, MDCXCV.

partly because of the interest naturally excited by the present war, but chiefly because of the new light that certain letters, hitherto unpublished, throw upon the life and character of a mathematician of merit, that this article is written.

After Wallis (1616–1703) had received the bachelor's degree at Emmanuel College in 1637 and the master's degree three years later, he became chaplain (1642) to the widowed Lady Vere. While so engaged there was brought to him a letter in cypher relating to the capture of Chichester.* In two hours he succeeded in deciphering it, and thus began his career as an expert in cryptography. As a reward for his services in this line he was given (1643) the living of St. Gabriel in London, and later (1647) that of St. Martin. His Savilian professorship (1649), his office of Keeper of the University Archives, and his chaplaincy to the king (1660), did not interfere with his work in the field of secret writing, and such was his standing that he was employed by the Earl of Nottingham as decipherer for William III. Loyal to his country, he steadily refused the importunities of Leibnitz to reveal his methods, but he made them known to his own son,† who lacked, however, the father's genius in this as in other respects, and to his grandson, William Blencow.‡

Some years ago, in working among the early mathematical manuscripts in the British Museum, I came across a considerable number of letters written by Wallis, many of them relating to this work in cryptography. While it is not feasible to publish them all, it may not be without interest to give a few of those which reveal the character of Wallis the man as well as of Wallis the cryptographer.§

The loyalty of Wallis to his own country, and his desire to have the work which he had begun for the state continue after his death, are seen in the following letter:

For MR TILSON, at

Sir,

OXFORD, Mar. 20, 1700/1.

I am obliged to you (though unknown) for your favour in drawing up the Warrant to be signed by his

* Dec. 27, 1642.

† John Wallis, b. Dec. 26, 1650, B.A., of Trinity College, Oxford (1669), called to the bar in 1676.

‡ He was the son of Sir John Blencow (b. 1642) who married the daughter of Wallis in 1675.

§ These letters are in Add. MS. 32499, entitled "Letter Book of Dr. John Wallis, 1651–1701."

Majesty for a Pension of 100^{lb}. p^{ann}m. payable to me, in order to my instructing a young man (William Blencow, son of Mr Justice Blencow) in the Art of Deciphering.

Concerning which I have this further to suggest; viz. That though it was his Mat^{ies} chief design in that grant, partly to gratify me for services allready done, and partly to incourage my propagating of that Art, that it do not dy with me; I think yet he may be inclinable allso to incourage the young man to diligence therein; and perhaps may be graciously pleased to order the pension payable to me during my life, (which is not like to be long (being now in my 85th year), and thenceforth to the young man during his Majesties pleasure.

I would have suggested this to my Lord Godolphin, or Sr Stephen Fox, but that I have not the honour to be personally acquainted with either of them. But if this may not be obtained, then to have it pass as it is now drawn.

I may add this allso; That I have been solicted by Mijn Heer Leibnitz, more than once, in behalf of the Elector of Hanover: who is willing to send hither some young men, whom he desires I would instruct therein; leaving it to me to make my own proposals on what terms I would undertake it. To which I have returned answe^r, That I shal be ready, my selfe to serve his Electoral Highness if there be occasion: but the skill of doing it, being a curiosity which may be of use to my own Prince, I do not think it proper to send it abroad, without his Ma^{ties} leave.

If you shal think it proper (in such way as you may judge most fit) that this be layd before his Ma^{tie}, you may therein oblige,

Sr Your humble servant

JOHN WALLIS.

Of his painstaking work in deciphering, the following letter addressed to R. Hampden* gives evidence:

* Richard Hampden (1631–1695) was a son of John Hampden (1594–1643) of revolutionary fame. At this time he was representing Wendover and was prominent as a spokesman of the Whigs.

For R. HAMPDEN ESQ^r, at the house of Comons.

S^r

OXFORD, Aug. 3. 1689.

In a paquet from the Earl of Nottingham, I lately received inclosed a Letter from you to which (in a like paquet to his Lo^p. by the last post) I returned a short answer. But should have been more particular were it not for fear of loosing that Post; which was within a few minutes of going before I had dispatched that paquet as it was; (though I had taken true pains about it, both by day, & by night, because you desired to have it done speedily).

I find, in the Cipher, there had been some mistakes committed: As (in one place) *Brigadies* for *Brigadier* (where 36, had been miswritten for 35;) and *efget* for *effet* (twice;) and *pued* for *pied*; and *aller* for *aller*: All which (without altering the original, which I was not to do) I tooke the liberty to correct in my transcript. Beside which, I do not know that I have varied a letter from what was in Cypher, save that in one place, I doubt I have miswritten *pouvoit* for *pouvoit*.

Some few words I could onely supply from the sense; As (in the second or third line) *c s*, of which I was sure (that is, that it was a word which begins with *c*, to which *s* was added in the end) for which (from *y*^o sense), I supply *chances* (because I could not think of any other such word which would make good sense, the same figure occurs not again.) In like manner *sans*, *tout*, *homme*, (& some few other) but these are confirmed by their so occurring several times in different circumstances; of which therefore I make no doubt.

For some few (of which I had some doubt) I have left blanks; as (in the second line) *entraîne t* (where I think *train* or *traict* or some such word is to be supplied.*

Soon after, there is *c toute sorte de r* which is I presume to be read *contre toute sorte de regle*: which occurs again in *y*^o. next line, where *regle* is spelled by letters; unless in the former place we choose to read *raison*.

Some other French words I mett with, which I did not understand but I give them as I find them: & in some

* No; but tousiours [so in margin of original].

places there seems to me some (sic) a word wanting (which yet I would not take upon me to supply) as, where *donner* doth (two or three times) occur for *ordonner*; (to *order* or *give order*), but perhaps it is onely my being unacquainted with y.^e French way of speaking.

M^r d'Hamilton and *M^r Rose*, I do not know whether I am to read *Monsieur* or *Monseigneur* (because I do not know the quality of the persons;) But I think y.^e latter; because I find the same title afterwards given to the person to whom he writes, w^{ch} (as appears by y.^e beginning & end of the letter) must there be read *Monseigneur*. But *M. . . Melfort* (where another figure is used) is, I presume, to be read *My Lord Melfort*. But I chose to write *M^r d'Hamilton* &c because in the beginning of the letter (which is not in cypher) I find written *M^r d'Hamilton*.

Non estoit parfaitement b. . . I suppose is to be supplied by *bas* or some other such word beginning with *b*.

En e. . . de tout craindre, is to be supplied by *exces*, or some such word beginning with *e*.

Ny a. . . pas ici, is to be supplied *avons*.

L'escrit m. . . apres is to be supplied by *mesme*, or some other word beginning with *m*.

And, near the end, *e. . . e*, is certainly to be read *envoyé*, but the word not then coming to mind, I left a blank.

Of all the rest, I think myself very secure not to have missed any thing.

When he says, there were killed ore wounded, *about forty officers & some sixty soldiers*; it seems strange that, if 40 officers, there should not be more than 60 soldiers; unless *soixante* (60) be miswritten for *six cents* (600); but I give you as it is.

The name subscribed, I know not wheither it (sic) *Mointis* or *Nointis* or *Pointis*; For it is so written as it may bear any of them. I take it to be *Nointis*. But it is surely the commander in chief there.

Nor do I know to whom it is written (because the cover was not sent me,) but it must be somebody much superior to him that wrote it; as appears by the manner of writing.

Nor know I how it becomes to be so rotten and discoloured in so short a time: Unless possibly it may have been thrown overboard into salt water & recovered from thence.

I know not whether it may not be thought fit to have it translated into English & printed.

S^r. you will please to excuse my giving you. the trouble of so long a letter about such little things. But I was willing to let you see how far I could conclude certainly, & where but by conjecture; & upon what kind of grounds I did so conjecture. Those perhaps who are better acquainted with the language, may, in these cases, conjecture better, than

S^r

Your very humble servant

JOHN WALLIS.

Some idea of the importance of the communications that came into his hands may be obtained from a letter addressed to the Earl of Nottingham, then one of the secretaries of state with charge of the war department.* The letter is as follows:

For the EARL OF NOTTINGHAM at Whitehal.

My Lord,

OXFORD, Aug. 18, 1689.

I am almost ashamed to tell yo^r. Lordship how much time & pains & study I have employed on that very perplexed cypher in the Letter from Poland, & have not yet dispatched it. But by what I have done all ready, I find two things (which seem to me) of moment. One is a Treaty (or intreaty rather) of the French King with the King of Poland presently to make a war on Prussia.† The other, about a marriage of the Princess of Hanover with the Prince of Poland, promoted by the French King. How far it may be of concernment to us to know it, I am no competent Judge; But I had thought it did become me to give this timely notice of it (lest there might be a prejudice by delay) while I am preparing to give a fuller account of that letter (than yet I can), by the next Post (or that next after it) if I have not in the mean time order to come up with it myself. who am

My Lord Your honours very humble servant

JOHN WALLIS

* This was the second Earl of Nottingham (1647-1730), a great Tory but an anti-Jacobite.

† To give a diversion to the Emperour. [So in the original, as a note at the end of the letter.]

Many of the letters show Wallis as a man willing to serve the King but always seeking for rewards for himself and all his family. The following gives some idea of his methods of ingratiating himself with the powers at Court, a procedure then undoubtedly less objectionable in the eyes of the world than now:

To y^e EARL OF NOTTINGHAM at Whitehall.

My Lord,

OXFORD, Nov. 12, 1689.

I have ordered my son (who is now, or will be sudainly in London,) to wait on your Honour & acquaint you that I sent you a packet by last Sunday's Post; & two before by Sunday & Tuesday Posts next foregoing; which I hope came safe to yo^r Lo.^{ps} hand. But I am now at a stand, by reason that the five other letters are at lest in three (I think in four) different ciphers, & all new, which I have not had before. Besides which, I have myself (since my last) been very ill, so as not to be in a capacity to apply myself (as I would) to that or any other business which requires intentness; & continue stil indisposed. But I shal (as my strength & y^e difficulty of y^e work will permit) endeavour to serve his Majesty, & your Lordship therein.

I humbly thank yo^r Lordship for the kind expressions (of your readiness to oblige me) in yo.^r Lo.^{ps} letter of Octob. 24. I have not been fond of being a great man (studying more to be serviceable than to be great) & therefore have not sought after it. Especially in the late times, wherein I could safely foresee that divers things might then be expected from me, with which I could not comply. But I am capable of any promotion Ecclesiastical, or in a way of literature; & of my accomplishments for either, I had rather others should speak than myself. And I have a son, a Barrister of the Inner Temple who though his attendance on an estate he hath in the country have caused him to neglect the practise of y^e law. at London, yet he understands business very well, & is sagacious inough in it, & capable of any civil imployment; & of whom I could say more if he were not my son. And I hav a son-in-law (who married one of my daughters) M^r. Serjeant Blencow (of y^e. Inner Temple allso) an able Lawyer both at Chamber practise & the barr; & not inferior therein to divers of

those on the Bench; And neither of them in the lest wise sullied by any unhandsome actings in the late bad times; but are very cordial to his Ma^{ties} interest. Any thing of kindness to either of them, will be so, to

My Lord, Yo^r. Honours very humble servant

JOHN WALLIS.

Another letter, also showing the desire to be in the favor of the Court, and having some interest in view of the present European situation, is as follows:

To y^e EARL OF NOTTINGHAM at Whitehal.

My Lord, OXFORD Febr. 20, 1689/90. Thursday.

Before I left London (which was on Munday last) I was (on Saturday) to wait on yo^r. Honour both at Berkshire-house, & at y^e. Office in Whitehal, to receive your commands for Oxford, but was not so happy as to find you at either. And I had y^e same day waited on the Envoy of Brandenburg on the like occasion. Some while before which, a friend of mine dining at the Table of a Noble Lord, where divers Lords were present, & other persons of quality, one of them did openly declare, that y^e Elector of Brandenburg had sent me, for a present, a large medal, with an Inscription & a chain of gold (for deciphering French Letters &c). My friend sayd, He thought it was a mistake; for that he had spoken with me the night before, & I knew of no such thing. The other person of quality confirmed what he had before sayd, & went on to describe it so particularly as if he had seen it. Now (beside that this was not so) I doubt whether, if it were, it were advisable to make the thing so publike. That it was not so, I find, because I have since waited on that Envoye, & he sayd nothing to me to that purpose. He sayd indeed that his Master intended to send me a mark of his favour; but that none such was yet come; & added withal, that it is as the King's concern to gratify me for those services, as being a joint interest (sic) of him & his Allies. And, why I doubt whether it be advisable to make it so publike (that I decipher y^e French Kings letters) is, because it can then hardly be avoided but that this will some way or other come to y^e. French Kings knowledge from some of his correspondents here; which

will be attended at lest with one or both of these inconveniences, viz: A greater care to prevent the intercepting of such letters; And, a change of the ciphers they now use (which they have allready changed more than once, for I have allready nine or ten of their ciphers by mee) for others more difficult, & (perhaps) in superable. The two letters which I have brought with me from London, (one of two sheets, the other of five sheets) I shall endeavour to dispatch as soon as I get a little leisure. And am

My Lord, Yo^r. honours very humble servant

JOHN WALLIS.

That Wallis often felt that the rewards were not commensurate with the labor, and that he did not hesitate to make this fact known, may be seen from a letter written in 1691:

For M^r HARBORD, in St. James's park London.

Sir,

Aug. 15, 1691.

I should begin my letter with thanks for yo^r civilities of late, & before; And that you have pleased to concern yourself for me without my knowledge, (and without any complaint of mine.) But (because your time is precious) I shall rather obey yo^r commands, in giving you some account of what you ask me.

It is now more than two years compleat, since by a letter from my Lord Nottingham & another from M^r Hampden, I was desired to decipher an intercepted French letter, from y^e camp then before London-Derry; and (soon after) a second from y^e same place; &, then, a third from Poland to the French King from his Ambassador there. Which I performed to that satisfaction, y^t his Lordship made me a Present (from y^e King, I suppose) of Fifty pound. Which I looked upon as a handsome gratuity for y^e service then done, & as a testimon of his Ma^{ties} acceptance (which I valued) & returned my acknowledgements accordingly.

After this I was kept in a continual employment of like services: wherein I took true pains, (& my son with me, who was assistant to me;) by day & by night, (because letters came faster than we could well dispatch them:) And it was almost our whole employment for a long time.

About a year after; having received nothing (but good

words) in requital (since y^t for y^e first three letters) I let Mr Hampden know so much (upon who's joint desire I had undertaken y^e service.) Upon who's speaking(I suppose) to my Lo. Nottingham, he sent me another 50^l, which is all I have received.

Perhaps you will judge this to be no great gratuity for so much service (which every body could not have done him:) Having by that time deciphered some hundred sheets of paper, sent me in cipher, for that purpose. And some of them, (he can tell you,) were of very great concernment: The deciphering some of those letters having quite broke all y^e French King's measures in Poland for that time; & caused his Ambassadors to be thence thrust out with disgrace. Which one thing was of much greater advantage to his Ma^{ty} & his Allies, than all that I am like to receive on that account.

I remember that my Lord Arlington (a little before he gave over being Secretary) sent for me on purpose, & did (without my asking) give me 50 Guinny's in hand; & promised me 50 guinney's a quarter (which were duely payd me) to endeavour the deciphering of such letters as should be sent me from time to time. And I had of him (as I remember) 200 Guinney's before he gave over y^e place of secretary. Yet I had not done for my Lord Arlington the tenth part of that service I have done my Lord Nottingham.

However; without disputing the point, whether that Fifty pound were a Noble recompense for all y^e services then done (except y^e three first letters) & for all that I was to do afterward (for that, it seems, is now the case:) I have continued the same trade for more than another full year, without having received (or being like to receive, for ought I see) y^e value of one penny.

'Tis true; I have had all along a great many good words; That he is my humble servant; My faithful servant; my very faithful servant; That he will not fail to acquaint y^e King with my diligence & success in this difficult matter; That he is very sensible of y^e pains I had taken in this troublesome work; That he shal be very glad of any opportunity of doing me any service: That y^e King is sensible of my zeal & good affection, and will (he beleeves) in a short time give me some mark of his favour (meaning,

I suppose, some preferment,) wherein his Lo^{ps}. endeavour shal not be wanting; That he will not be wanting to represent my services to my advantage. And y^e like.

But these Promises are of Two-years standing. And it may seem strange; if in all that time (while so many Deans, & Bishops, & Arch Bishops have been made) his Lo^p. could never find an opportunity of what he so much desired.

Now, y^e Apothecaries tell us, that some Cordials are y^e less Operative, for having been long made. And I have been told (by one who pretends to understand them) that Courtiers promises, if they do not operate quickly, it is not likely y^t they will operate at all, (but are to pass for words of course:) and that to depend upon stale promises, is like calling for an Old Debt; w^{ch} y^e longer it hath been forborn, grows y^e more desperate.

As to what he intimates of preferment: I have asked none, & they offer none, (& so far we are agreed:) And there are so many who (by themselves or friends) seek preferment, y^t it is not like to fall upon one who looks not after it. And I easily foresee so many exceptions, & Diversions, that may be made by persons otherwise concerned, (as, y^t one thing is too good for me; another, not good enough; a third, fit for somebody else, y^t is to be first served: and y^e like:) that I have no great expectation of it (being not acquainted with y^e methods of Court applications) nor much concerned for it: Having allways endeavoured rather to deserve preferment, than to have it: I may perhaps, in time, be offered something that may not be advisable for me to embrace, (because I must, for it, leave what I have in Oxford; which if not better than it; may at least be so to me;) & then it will be easy to object that he who doth not accept a little one, doth not deserve a better.

But having (for my Lord Nottingham) condescended to do clarks-work; I might at least expect clarks-wages; (without being thought mercenary or ungentile:) And I presume there is never a clark his Lo^p. keeps, but is (one way or other) better payd, for y^e work he doth, than I am.

He may say perhaps, This is (not his, but) the King's service. Very true. And so is all y^e service his Lo^p. doth as Secretary. Yet he is well payd for it. And, so

wel, that he may (out of his allowance) afford to gratify those y^t work under him.

All which is sayd, not that I do distrust their Ma^{ties} gracious inclinations; or, would or (sic) under value their favours. But, because Princes see with other mens eyes; & know no more of Persons or services than as they be represented. And then, if one man (by a friend at Court, who hath the Kings ear) be represented as a person of prodigious parts, of great learning, of a clear reputation, of great esteem at home & abroad (in all parts of y^e learned world,) as a credit to y^e University, an honour to the Nation; as a friend to the Government, & zealous for it; of a good judgement, a clear apprehension; that speaks well & writes well, in what he undertakes, & may (if employed) be many ways serviceable to the Government; a man of temper & moderation: not bigotted to a faction (laying great weight upon little things, which cannot bear it, or deserve it;) who hath done, & can do services which others do not pretend to: (though there may be much of Hyperbole in all this:) And another be represented, as, An obscure person (not known at Court,) that hath a knack perhaps to decipher a letter, or so, (& may be made use of for such a purpose,) but makes no figure in the world, nor looks for any great matters; & may be easily induced (by a few good words) to do what you would have him: In such case, I say, it is not hard to judge, whether of the two stands fairest to be preferred.

Nor is it sayd, as if I were not willing to serve their Ma^{ties} without being Hired. But the quite contrary. I have been, & shal be willing to serve their Ma^{ties} the best I can upon all occasions. And to do it gratis (if it be their pleasure it should be so:) & do it as honestly as those who are better payd. Of their Ma^{ties} I make no complaint at all. All we can do for them is little inough: & it is impossible they can gratify all.

But when I am employed (in troublesome work) by persons of Honour: I expect they should do honourable things: And, if therein I am mistaken, it is their fault, not mine. I am

S^r your very humble servant

JOHN WALLIS.

Out of the mass of letters on the subject, of which I have copies, a single example may be given in conclusion to show both certain mental characteristics of Wallis and also something of his elementary work with ciphers. The letter was written to James Johnston* whom Swift called "one of the greatest knaves, even in Scotland."

To M^r JOHNSTON Ambassador to Berlin.

S^r June 9, 1692. SOUNDESS BY NETTLEBED,
OXFORDSHIRE.

Your letter to me of May 28. came to Oxford at a time when I was not there; nor have I been there since, but it was sent after me where I now am. I thank you for the kindness you did me, or would have done me, at Berlin. And if your endeavours did not succeed, I am not to blame you for it. Onely I could have wished, you had told y^e Elector himself that I had yet received nothing (& he might then use his discretion,) rather than to Mons^r Smetteau who knew it well before. Of whom you will give me leave to say, that he might perhaps act like a Statesman, but certainly not like a Gentleman; to treat me like a child, as if I were to be wheedled on to difficult services by a few fair words, & a promise of sugar-plums, which should in the issue signify nothing. I did not want his present, (& perhaps can spare it as well as he,) nor did I ask any; till he told me of his own accord that his Master took my service very kindly, that he intended me an honorable present, & had given him orders to tell me so, & he expected it by y^e next Post. Nor did he onely say so to me, but caused it to be spread abroad (for I heard it from divers hands) that y^e Present was come, that it was a rich medal with an honorable inscription, & a gold chain of a great value. Now if such present was sent me, but never came to my hand; I was wronged on a double account: first in missing the present that was intended me; & then in being thought Rude by his Electoral Highness for not returning some acknowledgement for the bounty which he thought I had received. But if no such present was sent, nor was intended; He should have said nothing of it, & I should have said as little. Yet when

* James Johnston (1643?-1737), son of Sir Archibald who was hanged for treason (1663) by order of Charles II.

I had deciphered for them between two & three hundred sheets of very difficult, & very different cipher, they might at least have afforded me Porters pay; if not that of a Scrivener. Yet I would not be thought in this to reflect on his Electoral Highness, but on Myn-heer Smetteau. For Princes know no more of Things or Persons, in such cases, but as they be represented by their Ministers. His last evasion, of y^e Coyners slowness, is so pittyfull an excuse that I am ashamed to hear it. For if a Coyner cannot, in three years, prepare a Medal; a Goldsmith certainly might, in a less time make a Gold-chain; (or somewhat equivalent:) or if he had remitted hither the value in Mony, it might have been made at London. But inough of this.

As to what you desire, concerning an easy Cipher, which yet may be tolerably safe, because you are to use it yourself & cannot well spare much time for it: I think you may venture on such as this,

For y^e letters, a b c d e f g h i k l m n o p q r s t u
w x y z, put the next after them, b c d e f g h i k l m
n o p q r s t u w x y z a, or the next before them, z a b c
d e f g h i k l m n o p q r s t u w x y,

or the next but one, or two, before or after them in the Alphabet; and intermix (as Nulls) the numeral figures, 1 2 3 4 5 6 7 8 9 or the greek letters $\alpha \beta \gamma \delta \epsilon \zeta$ &c, or what other such notes you please. (And you may by one such alphabet correspond with one person, & by another with another whereof neither shal understand the other's cipher. (Which is a method very ready for use; (And I know that K. James with his correspondents here doth use Ciphers not better than these.)) And 'tis tolerably safe, on these accounts. 1. 'Tis a chance, whether or no the letter be intercepted. 2. If so, 'tis yet a further chance whether those who have it will attempt its being deciphered (& not rather neglect it as thinking it to no purpose.) 3. 'Tis a further chance whether they light upon one who can do it. 4. If deciphered 'tis yet a chance whether it do you any considerable mischief; (for I presume you will not intrust any thing of very great moment therein.) And of all these casualties if any one so happen as you would have them, you are safe. If you desire one which (with a little more trouble) may be

somewhat more safe, I may send you one from Oxford
(whither I am now going) from

Sr your humble servant,

These,

JOHN WALLIS.

For Mr J. JOHNSTON, Secretary for Scotland;

at my Lord Nottingham's Office, at Whitehall,
London.

SHORTER NOTICES.

Analytic Geometry and Calculus. By FREDERICK S. WOODS
and FREDERICK H. BAILEY. Ginn and Company, 1917.
516 pp.

THIS book is a revision and abridgment of the authors' Course in Mathematics for Students of Engineering and Applied Science. In making this abridgment the authors have omitted determinants, theory of equations, poles and polars, diameters, center of curvature, special methods of integration, and complex numbers.

The first eight chapters deal entirely with analytic geometry and give the subjects usually given in a first course in American colleges. The following topics are exceptionally well treated: "Variables and functions," "Graphs" and the derivations of the standard equations of the conics. In several places the authors are very careless about a theorem and its converse, i. e., they prove a theorem and then state or use its converse. An example of this is on page 61 where they prove that two perpendicular lines have their slopes negative reciprocals of each other and then conclude that "two lines are perpendicular when the slope of one is the negative reciprocal of the other." Moreover no attention is called to the fact that if the lines are perpendicular to the coordinate axes their slopes are not negative reciprocals.

In Chapter IX they introduce calculus by means of slope and area. This is a very well written introduction to the calculus except for the fact that the definition of limit on page 130 is incorrect (the word "numerically" should be inserted after the word "remains" on line 4). Then follow chapters on the conventional work of maximum and minimum, tangents