This process may evidently be continued. We may then state the following

Theorem: The rth polar of B with respect to $C_n$ is $C_{n-r}$.

II.

Again let there be three distinct points $A$, $B$, and $C$ on the same straight line $l$, and through the point $C$ let the line $l_1$ be drawn perpendicular to $l$. Let lines $l_2$ and $l_3$ be drawn through $A$ and $B$ respectively, and let $l_2$ and $l_3$ intersect on $l$. Let $l_2$ make an angle $\alpha$ with $l$, and $l_3$ make an angle $\beta$ with $l$, and let a line $l_4$ be drawn through $B$, making an angle $\eta\beta$ with $l$. Let $l_2$ and $l_4$ intersect in $D$. Then just as in section I, the equation representing the locus of $D$ is

$$k \left[ x^n - \binom{n}{2} x^{n-2} y^2 + \cdots \right]$$

$$= (x - c) \left[ \binom{n}{1} x^{n-1} - \binom{n}{3} x^{n-3} y^2 + \cdots \right],$$

where $k = (a - c)/a$ and $a = AC$, and $c = AB$.

It is then evident that the theorem in section I holds for the curve represented by equation (7).

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ON THE RECTIFIABILITY OF A TWISTED CUBIC.

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Given the twisted cubic

$$x_1 = at, \quad x_2 = bt^2, \quad x_3 = ct^3, \quad abc \neq 0;$$

to show that the condition that it is a helix is precisely the condition that it is algebraically rectifiable.

If (1) is a helix, then $T/R$, the ratio of curvature to torsion, is constant. Denoting differentiation with respect to $t$ by
primes, we have
\begin{align*}
x' & : a \quad 2bt \quad 3ct^2, \\
x'' & : 0 \quad 2b \quad 6ct, \\
x'''' & : 0 \quad 0 \quad 6c,
\end{align*}
\begin{align*}
(x' | x') &= a^2 + 4b^2t^2 + 9c^2t^4, \\
(x'' | x'') &= 4(b^2 + 9c^2t^2), \\
|x'x''x''''| &= 12abc,
\end{align*}
\begin{align*}
(x'x'' | x'x''') &= (x' | x')(x'' | x'') - (x' | x'')^2 \\
&= 4(a^2b^2 + 9a^2c^2t^2 + 9b^2c^2t^4).
\end{align*}
\[T/R = -\left(\frac{x'x'''}{x'x'''}\right)^{3/2}\frac{1}{|x'x''x'''|}.
\]
Since \(|x'x''x'''|\) is constant, \(T/R\) is constant when and only when \((x'x'' | x'x''')(x' | x'')\) is constant. We thus have
\[4(a^2b^2 + 9a^2c^2t^2 + 9b^2c^2t^4) = \rho(a^2 + 4b^2t^2 + 9c^2t^4);\]
hence \(\rho = 4b^2\) and \(9a^2c^2 - 4b^4 = 0\). Conversely, for all values of \(a, b, c\), \(abc \neq 0\), for which
\[9a^2c^2 - 4b^4 = 0,
\]
\(T/R\) is constant—in particular, is equal to \(\mp 1\), according as \(2b^2 = \pm 3ac\)—and the cubic (1) is a helix.

If we had fixed our attention on another characteristic property of a helix, namely, that the tangent makes with a fixed direction a constant angle, we should have again derived the condition (2). The fixed direction—that of the axis of the cylinder on which the helix lies—is \((1/\sqrt{2}, 0, \pm 1/\sqrt{2})\) and the helix cuts the rulings of the cylinder under an angle of 45°.

That (2) is a necessary and sufficient condition that \(s\), the arc of (1), is an algebraic function of \(t\) and hence that (1) is algebraically rectifiable follows from the fact that the integral
\[s = \int_{b_0}^t \sqrt{a^2 + 4b^2t^2 + 9c^2t^4}dt
\]
is algebraic when and only when (2) holds. Hence the theorem: The twisted cubic (1) is algebraically rectifiable when and only when it is a helix.