

SHORTER NOTICES

Lehrbuch der darstellenden Geometrie. By Dr. Georg Scheffers. Vol. I, ix + 423 pages, 404 figures. Vol. II, xiii + 439 pages, 396 figures. Berlin, Julius Springer, 1919, 1920.

This treatise on descriptive geometry contains in the main the substance of the lectures given by Professor Scheffers at the Technical School of Berlin. It presupposes nothing but "school-mathematics" on the part of the student, which, in America, is approximately equivalent to mathematical training in college algebra, trigonometry, and analytic geometry.

The courses in descriptive geometry in the universities and polytechnic schools of continental Europe are usually distinguished by high scientific standards and are given by men who are either mathematicians, like Scheffers, or men who have at least considerable mathematical knowledge beyond the calculus. The type of courses in descriptive geometry offered in most of the American engineering schools, serving a purely utilitarian purpose, may be found in the European trade-schools and secondary technical schools of various kinds, where, naturally, no effort is made to treat the subject as a science, to place connecting links with other branches of mathematics, or to pay any attention to historical developments.

From the standpoint of the scientific critic Scheffers' *Lehrbuch* is a masterly exposition of descriptive geometry. Almost on every page one may notice the enormous advantage in the treatment of the subject by the superior insight which a deeper mathematical knowledge affords. Nevertheless, Scheffers, even from the practical standpoint of the engineer, never grows pedantic, or loses himself in some individual pet schemes.

The concepts and propositions of projective geometry are developed as far as they are of unquestionable value in the treatment and rational solution of certain characteristic problems. Any one, like Scheffers, who is familiar with modern progressive views will, of course, contend that it is impossible to write a modern scientific descriptive geometry without the knowledge of projective geometry, even when taken in the larger sense of the word. The frequent lack of such knowledge on the part of teachers and writers accounts for many obsolete methods of treatment and the omission of some of the most beautiful applications of projective geometry to descriptive geometry.

The stimulating influence by Scheffers' competent selection and presentation of the subject-matter upon the reader is increased by extensive historical references and comments on geometric topics allied to descriptive and projective geometry. Throughout the treatise the author shows that he is well informed on the essential and important phases of the modern development of constructive geometric methods.

For didactic reasons, explained in the preface, the first volume starts with orthographic projection upon a single plane. The claim of modernity which the reviewer makes for Scheffers' book may, outwardly at least, appear from the fact that at the outset we find a statement about the principle of duality and its bearing upon projective processes. Further on we

find applications to the construction of roof-shaped surfaces, of sun-dials, of regular polyhedrons and crystal-forms, of ellipses and of moment-ellipses in graphic statics. The second chapter, which deals with parallel-projection upon a single plane, includes sections on cavalier-perspective, shadows under parallel light-rays, affinity and its applications, on the foundations of general axonometry (including, of course, Pohlke's famous theorem), on the theory of involution and its applications to the polar properties of the circle and the ellipse. In the third chapter we find the customary orthographic projection upon two or more planes of projection, including a valuable section on conics. Whenever possible, full use is made of the advantages which the application of projective properties of conics and the principle of affinity afford in the graphic representation of geometric forms.

The second volume contains two chapters on perspective and various applications. Chapter IV, on central projection, treats of fundamental concepts, so-called restricted perspective, construction of shadows in perspective, invariance of cross ratio, involutory perspective, applications of perspective, perspective of circle and sphere, properties of conic sections and their applications, including Pascal's and Brianchon's theorems, and so-called free perspective.

Various applications and supplementary topics, such as plane curves, surface-ornaments, topographical surfaces, surfaces of revolution, helical and cycloidal curves and surfaces, ruled surfaces, interpenetrations and shadows, and finally a brief account of relief-perspective, form the contents of the concluding fifth chapter.

The level upon which Scheffers proceeds may be judged from the fact that even a discussion of Peano's surface is included, to show the student the danger of hasty generalizations. Peano's original surface has the form $z = (y^2 - 2px)(y^2 - 2qx)$ in which p and q are positive real integers. For the sake of convenient constructive treatment, Scheffers discusses the projectively equivalent surface

$$z = -\frac{1}{10}(x^2 - 5y)(x^2 - y).$$

Every plane through the z -axis cuts the surface in a quartic which has a maximum at the origin, so that one might expect a maximum for the surface at that point. Still it is possible to trace curves on the surface passing through O having a minimum at O .

The whole treatise is carefully written and is typographically faultless. It may be heartily recommended to teachers as well as to students of descriptive geometry.

ARNOLD EMCH.

Girolamo Saccheri's Euclides Vindicatus, edited and translated by George Bruce Halsted. Chicago, The Open Court Publishing Company, 1920. 30 + 246 pp.

The original title of Saccheri's now famous work is: *Euclides ab omni naevo vindicatus*, . . ., which appeared in Milan in 1733, and which Halsted translates as "Euclid freed of every fleck." In English *fleck* sounds rather Teutonic, and the reviewer suspects that *flaw*, or *blemish* would sound better to the American ear. It is highly commendable that