

Introduction au Calcul Tensoriel et au Calcul Différentiel Absolu. By G. Juvet, with a preface by J. Hadamard. Paris, A. Blanchard, 1922. ii + 101 pp.

This small volume contains a very intelligible account of the so-called "Tensor Theory" which has come to the front with the Einstein relativity. The author starts in with elementary notions of vectors and their linear transformations, then defines tensors algebraically and in connection with bilinear forms, then geometrically. Some consideration of the elementary analysis follows, closing with an exposition of the "parallel displacement" of Levi-Civita, and a general discussion of tensors in a Riemann continuum. There is a good bibliography at the end.

The general point of view is set forth in the interesting preface of Hadamard. His thought reverts back to Poincaré, who said: "The physicist proposes problems to us whose solution he expects. But in making the proposal he pays for the most part in advance for our services. . . . There is an infinite multitude of combinations that may be formed of numbers and symbols. How should one select from this multitude those worthy of attention? Should we be guided by caprice? Such caprice would no doubt alienate our interests far and we would soon cease to understand one another. . . . But there is another side to the matter. . . . Physics not only prevents us from getting lost, but it prevents us from a more serious danger: that of wandering around in a circle."

This situation, Hadamard intimates, had actually arisen in infinitesimal geometry. In fact a crisis had arisen, which fortunately the relativity theory has resolved, through its stimulation of the study of methods already in existence, but mostly ignored, dating back to the absolute geometry of Ricci and Levi-Civita (*MATHEMATISCHE ANNALEN*, vol. 54).

It is a matter of satisfaction to those of us who have been interested in so-called vector methods, for some time, that they are finally coming into their own. The essential basis of such methods is not the avoidance of coordinates but the development of formulas which are intrinsically given. The present work is open to the criticism to which practically all these investigations are subject, namely that while expressions are produced which are invariant under transformations of the coordinates, there should be no use made of coordinates at all. A proper use of vectors makes the study of absolute geometry not only much simpler and almost obvious, but these same expressions may be translated directly into any desirable system of coordinates with little trouble. The invariancy is a direct consequence of the method. The author remarks in his introduction: "Systems of coordinates are not rejected, and in place of being foreign to the things studied, actually form their structure." Such a point of view is the common one, it is freely admitted, but it is to the detriment of the things studied. It merely shows the path of least resistance taken by the minds of the investigators. When a new generation think as readily in general vectors as many now do in ordinary vectors of three-dimensional space, instead of long demonstrations of invariant and covariant forms, occupying many pages, there will be a few pages of direct statements,

which reach the heart of the matter at once. The real criticism on the use of vectors is that they practically demand that the curved space under consideration be embedded in a flat space of a requisite number of dimensions. This difficulty, however, seems to exist as well for those methods that insist on coordinates. The work of M. Juvet is an example. A perfect intrinsic treatment would never leave the space itself. This can be accomplished by a properly developed system of general vectors. However, when one realizes that if the curved space is embedded in a flat space, then every formula in terms of vectors in the flat space relating to the curved space is *ipso facto* a covariant formula, he will see the whole matter of "theory of tensors" in the proper light.

Hadamard is certainly correct in taking the position that whatever the value of the relativity theory may be, it has done a great thing for geometry in opening up a new life to it, not of the momentary character of a new attack by some geometer, but of the permanent character infused by nature herself. Not only theorems in physics should be stated without systems of axes for reference, but theorems in geometry should also be so stated. The present work will be welcome to those who do think in coordinates, for its clarity and simple presentation.

JAMES BYRNIE SHAW.

Annuaire du Bureau des Longitudes pour l'An 1922. Paris, Gauthier-Villars. 1922. 7 + 800 pp.

There is little to say concerning the current issue of the ANNUAIRE. The review copy arrived somewhat late in the year but this matters less than might be imagined for an almanac, since the data, chiefly astronomical, which change from year to year, are always given in the almanac of the previous year.

There is an attractive little article on Relativity by E. Picard which sets forth the principal points of the theory and the astronomical tests. An article on Money and Exchange by Ch. Lallemand explains the fundamental bases of past and present currencies, and shows the fluctuations of their gold values in some detail during the past eight years.

E. W. BROWN.

Kartenkunde. By M. Groll. Neu bearbeitet von Dr. Otto Graf. I. *Die Projektionen.* Berlin und Leipzig, Vereinigung wissenschaftlicher Verleger, 1922. 116 pp.

This short treatise on map-projections is No. 30 of the well known Sammlung Göschen, and gives a fairly complete account of the numerous systems of mapping of the terrestrial globe. The introduction, which is concerned with general information about drawing, scales, and drawing instruments, and physical geography, is succeeded by four chapters dealing with various projections of the spherical surface upon a plane. In the fifth and last chapter we find a valuable summary and illustrations of the various methods of mapping in use and an historical sketch of their development.

On the whole the little book will be appreciated by students who wish to acquire a general knowledge of map-projections without spending