CONCERNING RELATIVELY UNIFORM CONVERGENCE*

BY R. L. MOORE

According to E. H. Moore, a sequence of functions \( f_1(p), f_2(p), f_3(p), \ldots \), defined on a range \( K \), is said to converge, to a function \( f(p) \), relatively uniformly with respect to the scale function \( s(p) \) if, for every positive number \( e \), there exists a positive number \( \delta_e \) such that if \( n > \delta_e \) then, for every \( p \) which belongs to \( K \), \(|f_n(p) - f(p)| < e \cdot s(p)|\).

In this note I will establish the following theorem.

**Theorem.** If \( S \) is a convergent sequence of measurable functions \( f_1(x), f_2(x), f_3(x), \ldots \) defined on a measurable point set \( E \) and \( S \) converges for each \( x \) belonging to \( E \), then \( E \) contains a subset \( E_0 \) of measure zero such that the sequence \( S \) converges relatively uniformly for all values of \( x \) on the range \( E - E_0 \).

**Proof.** Suppose that \( S \) converges on \( E \) to the limit function \( f(x) \). By a theorem due to Egoroff\(^\dagger\), \( E \) contains a subset \( E_1 \) of measure less than 1 such that \( S \) converges to \( f(x) \) uniformly on \( E - E_1 \). Similarly \( E_1 \) contains a subset \( E_2 \) of measure less than \( 1/2 \) such that \( S \) converges to \( f(x) \) uniformly on \( E_1 - E_2 \). Continue this process thus obtaining a sequence of point sets \( E_1, E_2, E_3, \ldots \) such that, for each \( n \), (1) the measure of \( E_n \) is less than \( 1/n \), (2) \( E_{n+1} \) is a subset of \( E_n \), (3) \( S \) converges uniformly on \( E_n - E_{n+1} \). Let \( E_0 \) denote the set of points common to the sets \( E_1, E_2, E_3, \ldots \). The set \( E_0 \) is either vacuous or of measure 0. Furthermore

\[ E = E_0 + (E - E_1) + (E_1 - E_2) + \cdots. \]

Since \( S \) converges uniformly on each point set of the countable collection \( E - E_1, E_1 - E_2, E_2 - E_3, \ldots \), it

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\( \dagger \) *Comptes Rendus*, Jan. 30, 1911.
follows, by a theorem due to E. W. Chittenden, that* $S$ converges relatively uniformly on the sum of all the point sets of this collection. But this sum is $E = E_0$.

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THE THEOREY OF CLOSURE OF TCHEBYCHEFF POLYNOMIALS FOR AN INFINITE INTERVAL†

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1. The Theorem of Closure. Suppose we have a function $p(x)$, not negative in a given interval $(a, b)$, for which all the integrals

$$\int_a^b p(x)x^n dx, \quad (n = 0, 1, 2, \ldots)$$

exist. It is well known that we can form a normal and orthogonal system of polynomials

$$g_n(x) = a_n x^n + \cdots, \quad a_n > 0, \quad (n = 0, 1, 2, \ldots),$$

uniquely determined by means of the relations

$$\int_a^b p(x)g_m(x)g_n(x)dx = \begin{cases} 0, & m \neq n, \\ 1, & m = n. \end{cases}$$

We call these polynomials Tchebycheff polynomials corresponding to the interval $(a, b)$ with the characteristic function $p(x)$. The simplest example is given by Legendre polynomials, corresponding to the interval $(-1, +1)$ with $p(x) = 1$.

The most important application of Tchebycheff polynomials is their use in the development of functions into


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