WHITE ON CUBIC CURVES

Plane Curves of the Third Order. By H. S. White. Published with the cooperation of the National Research Council. Cambridge, Harvard University Press, 1925. xii+168 pp. $2.75.

The author's purpose in writing this book clearly has been to produce, not a detailed treatise on cubic curves, but a clear, readable, and rigorous textbook explaining the outstanding projective properties of these curves. Thus his fundamental problem has been throughout, to select judiciously from a wealth of available material, and to present skillfully, the main features of the theory of cubic curves. In this he has succeeded admirably. The style is nowhere hurried, and the proofs are painstakingly thorough; yet the essentials of the subject are presented in a volume that is by no means lengthy. The author hopes that this work "may be found helpful as a stepping-stone to many extensive and beautiful treatises on special themes." For this purpose a considerable number of references are given.

The book starts with a study of the properties of real cubic curves invariant under real projective transformations. This is followed by a discussion of the simplest theorems on the intersections of cubics with other curves, especially with straight lines. There follows a discussion of considerable length on the properties of the chief invariant loci. Symbolic notation is here introduced and used, but the elaborate formal machinery of this notation is, as far as possible, deliberately and skillfully avoided. Several methods of generating the general cubic are then given, after which the problem of geometry on a cubic is approached, in the most natural and logical manner, by establishing and using Noether's fundamental theorem. This is followed by the parametric representation of the non-singular cubic by means of elliptic functions, with various applications showing the power of this representation. The final chapters bring in the notion of apolarity and point out some of the consequences of this theory.

One who wishes to secure as easily as possible an insight into the spirit and the methods of the projective theory of higher plane curves will find that a preliminary study of cubic curves constitutes an excellent introduction to the field. Most of the principal theories and modes of reasoning are duly exemplified, but in a preliminary and simplified form. The irrationality of the non-singular cubic, for example, introduces difficulties that can be mastered *simply* and rigorously by the use of Noether's $Af+B\phi$ theorem for cubics and the parametric representation of the curve by means of elliptic functions,—concepts both of which the student will find to be of fundamental importance in more advanced work. For an introduction of this character, the text under consideration is conspicuously well adapted. The presentation of the various topics makes full use of the peculiarities of cubic curves, yet the reasoning involved is of the type that the reader will need in the more difficult study of curves of higher order

The classical discussions of cubic curves in such texts as Salmon, Clebsch-Lindemann, Durege, and Schroeter, were excellent in their time, but have now become somewhat antiquated. In none of them do we find so wise a selection of material nor so balanced an emphasis on the various topics, as in this brief yet comprehensive and readable text.

C. H. SISAM

REYMOND'S HISTORY OF SCIENCE

Histoire des Sciences Exactes et Naturelles dans l'Antiquité Gréco-Romaine, by Arnold Reymond. Paris, Librairie Scientifique Albert Blanchard, 1924. vii+238 pp.

The Preface of this book is from the pen of Léon Brunschvicg, and is followed by an Introduction giving an outline of Babylonian and Egyptian science. The first three chapters are an historical survey of Greek and Roman science. The last six chapters deal with principles and methods in mathematics, astronomy, mechanics, chemistry, natural history, and medicine, as developed in Greece and Rome. In various places, there is pointed out to the reader the connection of ancient conceptions in science with those of later periods.

The work under review is not an independent research based upon the study of original sources, but a compilation from European publicatons. Of assistance to the studious reader are the six pages given to bibliography. Our assurance that all important publications are included in the list is shaken somewhat by the omission of all reference to G. Eneström and his BIBLIOTHECA MATHEMATICA. In the case of the Moscow papyrus, the author overlooked the all-important early Egyptian computation of the volume of the frustrum of a quadrangular pyramid.* Except for a reference to an article of J. H. Breasted that was published in Europe, American scholarship is ignored completely. For a general view point of Babylonian and Egyptian mathematics, a reference to L. C. Karpinski† would have been of value. R. C. Archibald's reconstruction of Euclid's *Divisions of Figures*‡ escaped the attention of the author, as did also D. E. Smith's choice article§ on Greek computation. American publications would have afforded the author a profounder realization of the importance in the history of the theory of limits of Greek discussion of "Indivisibles" and of Zeno's arguments on motion.|| Of interest would have been the British defence of Aristotle's treatment of falling bodies, to the effect that Aristotle dealt with terminal velocities of a body falling through a resisting medium,¶ and a

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* ANCIENT EGYPT, vol. 17, p. 100.

† AMERICAN MATHEMATICAL MONTHLY, vol. 24 (1917), pp. 257–265.

‡ R. C. Archibald, *Euclid's Book on Divisions of Figures*, 1915.

§ D. E. Smith, BIBLIOTHECA MATHEMATICA, (3), vol. 9 (1908–09), pp. 193–195.

|| F. Cajori, AMERICAN MATHEMATICAL MONTHLY, vol. 22 (1915), pp. 1, 39, 77, 109, 143, 179, 215, 253, 292.

¶ NATURE, vol. 92 (1914), pp. 584, 585, 606.