SHORTER NOTICES

A Course of Differential Geometry. By J. E. Campbell. Oxford, Oxford University Press, 1926. xv+261 pp.

The manuscript for this book was completed by Professor Campbell shortly before his death in 1924. It was prepared for the press by Professor E. B. Elliott. The book was evidently planned to prepare the reader to understand differential geometry as applied in modern relativity theory. The methods of tensor analysis have been employed throughout.

The first chapter gives a brief but quite satisfactory introduction to the tensor theory associated with a symmetric quadratic differential form with n independent variables. The Christoffel symbols of three and four indices are defined and discussed and the operation of covariant differentiation is introduced without being so called. At the end of this chapter an Einstein space is defined as one in which certain tensor components vanish but no explanation of the definition is given at this point.

The next nine chapters contain a treatment of differential geometry of ordinary space, tensor methods being used almost exclusively. Particular attention is given to the following subjects: equivalence of forms, geodesics, curvature deformation of surfaces, congruences, curves in space and on a surface, ruled surfaces, minimal surfaces, conformal representation, orthogonal surfaces.

In order to understand these nine chapters the reader must have previously acquired the main facts of differential geometry. Little explanation outside of the analysis is offered and the geometrical concepts are usually introduced without definition. The purposes of this part of the book undoubtedly are to show the power of vector methods and to prepare the way for extensions to hyperspace.

In the last four chapters we return to the consideration of the quadratic form with n independent variables and are thus led to geometry in n-way space. As a generalization of the straight line in the plane we have the geodesic. Gauss's measure of curvature has as a natural extension Riemann's measure of curvature, in which a tangential n-fold takes the place of the tangent plane.

The fundamental form for an $(n+1)$-way space may be taken as

$$\phi du^2 + b_{ik}\, dx_i\, dx_k \qquad (i, k = 1, \cdots, n),$$

where ϕ, b_{ik} are functions of $x_1, \cdots, x_n$ and u. The surface $u=0$ is an arbitrary surface in the space. This $(n+1)$-way space is an Einstein space if for all such surfaces lying in it

$$\sum \frac{1}{R_i R_k} + \frac{1}{2} A = 0,$$

where the first term represents the sum of the products, taken two at a time, of the reciprocals of the principal radii of curvature of the surface, and where

A is a certain invariant dependent on the Christoffel symbols of four indices for the n-way space of the surface. For $n=2$ the above formula reduces immediately to Gauss's well known formula for the product of the reciprocals of the principal radii of curvature of a surface. The above definition is easily seen to be equivalent to that stated in the first chapter.

A given n-way space is always surrounded by an Einstein $(n+1)$-way space whose generation can be accomplished by infinitesimal methods. The Einstein space which surrounds the n-way space $u=0$ is stationary if ϕ and b_{ik} are independent of u.

The last two chapters of the book are given to the discussion of n-way space of constant Riemannian curvature and of n-way space as a locus in $(n+1)$-way space.

Professor Campbell planned to add an appendix treating of the relation of the contents of the book to the physics of Einstein. Such an appendix would undoubtedly have added greatly to the understanding of the physical meanings of some of the mathematical results.

There is a complete absence of references in the book. Important results are not stated in the form of theorems or made to stand out on the page in any way.

E. B. STOUFFER

Les Fondements des Mathématiques. By F. Gonseth. Paris, Blanchard, 1926.

This book, which commences with a preface by J. Hadamard, deals with the foundations of geometry and kinematics and the relation between mathematics and logic.

In the first few chapters a discussion is given of various sets of axioms for euclidean space, the axioms of Geiger being compared with those of Hilbert.

Following a chapter on the continuum there is next a short chapter on the compatibility and independence of the axioms of a system. The author then passes on to the construction of continua in which he makes use of Jordan curves, topological transformations and groups of movements. His program is expressed as follows: Topology should be constructed with the aid only of the axioms of order and continuity.

Chapter VI is devoted to non-euclidean geometry and the next chapter to the relation between theory and experience. The discussion of time and relativity is enlivened by a short dialogue in which Bergson, Metz, LeRoux, Fabre, and the author are supposed to take part.

There is next a chapter on the idea of motion and general relativity and the book closes with a chapter on mathematics and logic in which special attention is devoted to the vicious circle of Weyl, Brouwer's remarks on the principle of the excluded third, Weyl's formulation of the continuum and Hilbert's logical foundations of mathematics. There is also a short article on formal logic.

H. BATEMAN