PÓLYA AND SZEGÖ'S PROBLEMS IN ANALYSIS

Aufgaben und Lehrsätze aus der Analysis. By G. Pólya and G. Szegö. Berlin, Julius Springer, 1925. vol. 1, xvi+338 pp; vol. 2, x+407.

There are but few books which could be compared with this one as to the richness and charm of material, and amount of suggestions which an attentive reader is able to get out of it. The purpose of the book, and hence the material and its treatment, is quite different from those of numerous collections of problems already existing. The mere development of technique is of secondary importance with the authors: their main efforts are directed rather toward cultivating good habits in the mathematical thinking of their readers. The reader is constantly urged to give his attention not only to *what* he is being questioned about, but also to *how* and *where* he is questioned. Accordingly the most essential feature of the book, and one to which the authors gave much care, is the relative order of the problems. Isolated problems and examples comprise but a small part of the book: usually the reader has to deal with sets of problems, each of which is devoted to an independent and more or less substantial notion or question.

The material treated in the book (see the detailed list of contents below), is taken from the modern parts of the classical theory of functions. This choice seems to be the wisest, because it not only agrees with the personal taste of the authors, but it also is fitted for the purpose better than any other part of analysis: it gives an adequate idea of the modern development of the science without being so abstract as to scare a beginner, even an advanced one. As sources, the authors use to a great extent recent memoirs. Many a problem has been communicated to them by different mathematicians as well as those personally found, and appears in print for the first time. There is no doubt, therefore, that the book may be of great value to specialists as well as to beginners.

The first volume is devoted to the fundamental notions, while the second volume treats of questions of a more specialized type.

Volume I contains three sections:

Sec. I (Chapters 1–4) Infinite Series and Sequences.

Sec. II (Chapters 1–5) Integral Calculus.

Sec. III (Chapters 1–6) Generalities on Functions of a Complex Variable.

Of these: Chapters I, 3 (the Structure of Sequences and Series of Real Terms); II, 1 (The Definite Integral as a Limit of Sums of Rectangles); II, 3 (Inequalities); II, 4 (Different Kinds of Distribution, i. e. Multiples of an irrational number, distribution of digits in a logarithmic table, etc.); II, 5 (Functions of Large Numbers); III, 3 (Geometrisches über den Funktionsverlauf); III, 4 (Cauchy's Integral and the Principle of the Argument); III, 6 (The Principle of the Maximum) are of extraordinary interest and value.

Volume II contains six parts:
Sec. IV (Chapters 1–3) Functions of a Complex Variable, special part.
Sec. V (Chapters 1–3) Position of the Zeros.
Sec. VI Polynomials and Trigonometric Polynomials.
Sec. VII Determinants and Quadratic Forms.
Sec. VIII (Chapters 1–5) Theory of Numbers.
Sec. IX Geometric Problems.

This volume is still more interesting although harder than the first.

The readers will find considerable help in the solutions which are separated from the problems and collected in the second part of each volume. The solutions are given in a very condensed form, and require very careful reading. They are often followed by suggestions of possible generalizations and contain bibliographical references.

The total number of problems in the first volume is 747, and in the second volume 877.

It is the authors' teaching experience that each chapter can be worked through in an advanced class in one semester (2 hours per week); this certainly requires that the students be excellently prepared.

Both volumes are beautifully printed and like most of the monographs of this series by J. Springer (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen) seem to contain almost no misprints.

Of course nobody would expect that a book so rich in content and original ideas would be entirely free from a few minor defects. Let us indicate some of them which occurred to us at the first reading.

Problems I, 75, 76 should be placed before I, 72–74. Problem I, 113 should be placed after I, 139. The person who does not know the method of solving Problem II, 202, could hardly work out the solution of II, 58.

Solutions of some problems could be simpler and more natural (this point is very important, because we are interested not only in the facts as such, but also in the methods of obtaining these facts): this is the case with Problems I, 73–74, 82, 174.

Some references are not quite exact, for instance, the solution of Problem II, 199 is ascribed to P. Czillag, while actually it was published (in more general form) by C. N. Haskins in his well known paper *On the measurable bounds and the distribution of functional values of summable functions*, Transactions of this Society, vol. 17 (1916), p. 181–194.

Some topics, in view of their importance and relative simplicity, should be presented in greater detail than has been done in the *Aufgaben*. Such topics are, in our opinion: dominant series; summability of divergent series.

To conclude our review, let us modify one of the metaphors used by the authors in the Introduction: We feel as though we were in a large and prosperous city; our guides are successful in their intention to show us how to travel from any place to any other place in the shortest and surest way; their wide knowledge and fine taste enable them, meanwhile, to point out the places of unexpected and unusual beauty, and we are thankful for this.

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