RIETZ ON STATISTICS

Mathematical Statistics. By Henry Lewis Rietz. Chicago, Open Court, 1927. vii+181 pages.

This is Number Three of the Carus Mathematical Monographs of the Mathematical Association of America.* As such, it is designed to give an exposition "comprehensible not only to teachers and students specializing in mathematics, but also to scientific workers in other fields." The author defines his main purpose as "shifting the emphasis and point of view in the study of statistics in the direction of the consideration of the underlying theory" and introducing "some of the recent advances in mathematical statistics to a wider range of readers." It must readily appear to the careful reader of the book that Professor Rietz has handsomely achieved his own stated purpose as well as the general purpose of the Carus series. The book does render available, as does no other volume written in English which is known to the reviewer, the essentials of an introductory survey of the underlying mathematical theory which must receive increasing attention from specialists in statistics, if the widespread use of statistical method is to be systematically helpful. Moreover, Professor Rietz has developed this theory so skillfully that the "workers in other fields," provided only that they have a passing familiarity with the grammar of mathematics, can secure a satisfactory understanding of the points involved.

As a general basis for developing the concepts of statistics, the author prefers to regard probability as the limit (if it exists) of the relative frequency, and it is likely that most practicing statisticians will agree with him.† Quite regardless of the considerable helpfulness of the concept of *a priori* probability, there can be little doubt that the notion of probability generally arises in practical problems of statistics as an adjunct in describing—one almost says "explaining"—observed frequencies. The idea of *description* is again prominent in the author's remark concerning the utility of fitted theoretical frequency curves: "Indeed, the use of the theoretical curve is likely to be justified in a large way only when it facilitates the study of the properties of the class of distributions of which the given one is a random sample by enabling us to make use of the properties of a mathematical function $F(x)$ in establishing certain theoretical norms for the description of a class of actual distributions."

The author's definition of probability appears constantly in the background, particularly in Chapter II, entitled *Relative frequencies in simple*

* The first and second volumes of the series are: *Calculus of Variations*, by G. A. Bliss, 1925; and *Analytic Functions of a Complex Variable*, by D. R. Curtiss, 1926.

† His definition, to which he frankly says there are "some objections," is: "If the relative frequency of success approaches a limit when the trial is repeated indefinitely under the same set of circumstances, this limit is called the probability of success in one trial."

sampling, where he develops the probability integral and the Poisson exponential function. He says, for example: "The theorem of Bernoulli deals with the fundamental problem of the approach of the relative frequency m/s of success in s trials to the underlying constant probability p as s increases"; and again: "The De Moivre-Laplace theorem deals with the probability that the number of successes m in a set of s trials will fall within a certain conveniently assigned discrepancy d from the mathematical expectation sp." In connection with this latter citation, it may be remarked that the author seeks to make use of recent research in the history (as well as theory) of statistics. Thus, he names the theorem implying the "normal error curve" jointly for De Moivre and Laplace, instead of Gauss. The difficulties of substituting for the name of Gauss in the present extensive literature will be considerable, and not the least of these will be that of determining a name for the "Gaussian" curve. The word "normal" has such misleading implications that the substitution of the name "normal frequency function" does not seem entirely fitting.

Chapter III, *Frequency functions of one variable*, gives an exposition of the so-called Gaussian curve and of the Pearson and Gram-Charlier systems of curves. It is here that we find with especial frequency those strikingly helpful clarifying statements which feature all parts of the book. Thus, in referring to the "Gaussian" curve: "The difficulty is not one of deriving this function but rather one of establishing a high degree of probability that the hypotheses underlying the derivation are realized in relation to practical problems of statistics." Again: "The idea of obtaining a suitable basis for frequency curves in the probabilities given by terms of a hypergeometric series is the main principle which supports the Pearson curves as probability or frequency curves, rather than as mere graduation curves." And: "That is, the excess E is equal to the coefficient by which to multiply the ordinate at the centroid of the normal curve to get the increment to this ordinate as calculated by retaining the terms in $\phi^{(3)}(x)$ and $\phi^{(4)}(x)$ of the Type A series."

Correlation is the subject of Chapter IV, and the author presents the development both by use of the regression concept and by reference to the frequency surface.* In spite of the brevity of treatment, the topic of ordinary correlation is covered effectively; and the same may be said of multiple correlation.

The treatment of partial correlation deserves particular comment, especially in reference to the matter of definition. Professor Rietz suggests the possibility of regarding the partial correlation coefficient "as a sort of average value of the correlation coefficients of x_1 and x_2 in subdivisions of a population" in each of which the other variables have assigned values, or "as the correlation coefficient between the deviations of x_1 and x_2 from the corresponding predicted values given by their linear regression equations" on the other variables. Both of these notions are helpful, but the first is

* A typographical error appears in the section heading on page 78, and is cited because its prominent position may render it especially misleading. "Regressive" should read "regression."

especially useful in pointing out the nature of the error involved in the frequent definition of the partial correlation coefficient as that between two variables "when the other variables are held constant." It can not too strongly be emphasized that *no* variable is held *constant* in the partial correlation measurement. This process of measurement seeks merely to eliminate from the indicated correlation between two variables the part which arises from their mutual linear dependence upon certain other specified variables. The development in the book is especially happy in that it shows the only way in which constant or "assigned" values enter the concept of partial correlation.*

The remaining chapters, on *Random sampling fluctuations*, *The Lexis theory*, and *The development of the Gram-Charlier series*, are of somewhat less general interest than the earlier portions of the book. They are, however, important to the general reader; and the exposition of the probable error (standard error, rather than probable, is considered in the text) derivations and their significance is especially worthwhile.

The reviewer has not sought to verify in detail the symbolic portions of the text; but, if there be any mistakes, they are not such as to impede careful reading and full understanding of the discussion.

A difficult task has been handled by the author with admirable skill; and statisticians generally will be grateful for a volume which is available for courses and reference, for mathematicians and "other workers."

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PRINCIPIA: VOLUMES II AND III

Principia Mathematica. By Alfred North Whitehead and Bertrand Russell. Volumes II and III. Second Edition. Cambridge, University Press, 1927. xxxi+742 pp., and viii+491 pp.

The second and third volumes of the second edition of Whitehead and Russell's *Principia Mathematica* do not differ from the corresponding volumes of the first edition. An account of the changes which the authors think desirable is contained in the introduction and appendices to the first volume of the second edition, of which a review has previously appeared,† but the text of all three volumes has been left unchanged.

The second and third volumes of the *Principia Mathematica* are devoted to building up, on the basis of the system of logic developed in the first volume, the theories of cardinal numbers, relations and relation-numbers, series, well-ordered series and ordinal numbers, and finally of the continuum and of real numbers. The task of developing these theories on the basis of the theorems and processes of logic only, as well as that, undertaken in the first volume, of investigating logic itself by mathematical methods,

* In this connection, the reviewer regrets the parenthetical use (p. 101) of the words "held constant," although he is sure that no careful reader will be misled by them after going through the preceding discussion in the text.

† B. A. Bernstein, this Bulletin, vol. 32 (1926), pp. 711–713.