

*Vorlesungen über Algebra.* Fourth revised edition of the work with the same title by the late Dr. Gustav Bauer. By L. Bieberbach. Leipzig and Berlin, B. G. Teubner, 1928. x+334 pp.

The well known *Algebra* of Bauer\* now appears in a fourth edition remodeled and modernized to such an extent as to form practically a new book. In Section 1, on the fundamental properties of algebraic equations, we may note the thoroughly modern chapter on complex numbers, taken largely from volume 1 of Bieberbach's *Funktionentheorie*, and in Chapter 3, Cauchy's function-theoretic proof of the fundamental theorem of algebra is followed very appropriately by Rouché's theorem, which is used to prove the continuity of the roots as functions of the coefficients. Section 2 deals with determinants, matrices, linear equations, quadratic and bilinear forms, and the proofs are made short and elegant by an extensive use of the vector notation. Section 3 deals with symmetric functions and Tschirnhaus' and Jerrard's transformations in the customary manner. Section 4, which carries the too narrow title "numerical solution of equations," is the most distinctive feature of the book. The first chapter gives various methods for finding upper and lower bounds for the real roots, and then proceeds to Lill's graphical method, which is the most rapid means of obtaining fairly good approximations to the real roots; better approximations may then be obtained by the standard methods which are briefly described. It is pleasing to see less than half a page devoted to Horner's method instead of the inordinate amount of space given to it in American texts. Chapter 2 gives Descartes' rule of signs, and the theorems of Rolle, Budan Fourier, and Sturm, including Hermite's treatment of Sturm's problem by quadratic forms. Chapter 3 contains the method of Cohn (*Mathematische Zeitschrift*, vol. 14) for determining the number of complex roots inside a circle, theorems of Schur and Kakeya, and Schur's discussion of equations with all the roots having negative real parts.

Chapter 4 gives a thorough discussion of Graeffe's method both for real and complex roots. Chapter 5 begins with the classical theorem of Gauss on the relative position in the complex plane of the zeros of  $f(x)$  and  $f'(x)$ , proceeds to theorems of Poulain, Laguerre, Fejér, Grace, Heywood and Walsh, and also gives composition theorems due to Malo and Schur. The reviewer is particularly pleased to see the very readable account in Chapters 3 and 5 of a number of quite modern and interesting things, which are sadly neglected in most other texts.

Section 5, on the algebraic solution of equations, gives a very readable account, mostly along classical lines, of the elements of cyclotomy, substitution groups and the Galois theory; there is a new proof of the irreducibility of the cyclotomic equation due to Späth (*Mathematische Zeitschrift*, vol. 26). The book closes with an elementary account of continued fractions.

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\* Reviewed in this Bulletin: first edition, vol. 10 (1903-4), p. 257, by L. E. Dickson, and second edition, vol. 17 (1911), p. 428, by A. Dresden.