YOUNG ON PROJECTIVE GEOMETRY

Projective Geometry. By J. W. Young. Chicago, The Open Court Publishing Co., 1930. ix+185 pp.

This volume is the fourth in the series of Carus Mathematical Monographs sponsored by the Mathematical Association of America. The natural point of departure for the reviewer would seem to be the expressed purpose of the Carus Monographs, namely, "to contribute to the dissemination of mathematical knowledge by making accessible at nominal cost a series of expository presentations of the best thoughts and keenest researches in pure and applied mathematics," "in a manner comprehensible not only to teachers and students specializing in mathematics, but also to scientific workers in other fields, and especially to the wide circle of thoughtful people who, having a moderate acquaintance with elementary mathematics, wish to extend their knowledge without prolonged and critical study of the mathematical journals and treatises." For the present reviewer, this would mean to start with a handicap of skepticism. The "other scientific workers" are finding a steadily increasing need of mathematical training. Their first interests lying in other directions, they naturally wish to secure this training at the least possible cost of time and effort. Each individual will have to find out for himself the minimum expenditure for which he can acquire a useful tool. The Carus Monographs may be helpful to this group. But is it possible to make accessible the keenest researches in mathematics to that wide circle of thoughtful people who wish to extend their knowledge without prolonged and critical study? To ask a simpler question, is it possible to convey to a group of intelligent readers having a moderate acquaintance with elementary mathematics something of the beauty and fascination of the ideas of projective geometry, unless these readers are willing to pay the price of the moderately prolonged and critical study which the average mathematician has devoted to the subject? Whatever the answer to this question, the reviewer ventures the opinion that *if it can be done* Professor Young's book does it.

But if one may be allowed to forget the stated purpose of the Carus Monographs, he is then free to express unreserved approval of this excellent piece of expository writing. The usual first course in projective geometry (so-called) is a mixture of projective and metric geometry in which distance, perpendicularity, and circles are as prominent as harmonic sets, involutions, conics, and polar systems. Probably such courses are useful; but it is doubtful whether they leave the student with any clear conception of what is meant by *projective* geometry. And just at this point one could do the student no greater service than to place in his hands the little book under review.

After defining projective space by introducing the "ideal" elements, the first five chapters proceed to develop pure projective geometry by the synthetic method. The treatment calls upon the reader frequently to exercise his geometric intuitions. It is not at all logical in the sense of building up the subject from an explicitly stated set of postulates; nor is it illogical in any derogatory sense. With respect to this part of the book, one can readily make himself believe in the group of non-mathematical readers, and in their keen interest and pleasure as the subject unfolds itself. Starting with the idea of duality in the plane and in space, one is led on to the consideration successively of the four-point and four-line in the plane; the complete five-point in space; the Desargues figure as a plane section of the complete five-point in space; harmonic sets of points and lines; projectivities between one-dimensional forms, with the so-called fundamental theorem, and a treatment of double elements; involutions; and conics, with the theorems of Pascal and Brianchon and a treatment of poles and polars. From these eighty pages unmarred by any mention of distance, angle, or circle, a student may catch the real spirit of projective geometry.

In Chapter 6, on Metric Properties, there is clearly set forth the relation between metric and projective properties. It is refreshing to read an exposition in which ordinary metric geometry and projective geometry are *related* without being *confused*. Our school texts in euclidean geometry define parallel lines as lines lying in the same plane but not meeting in a point; teachers who had studied projective geometry began to tell high-school students that these parallel lines "meet in a point at infinity"; and then to avoid the rather bald contradiction they amend the definition of parallels (*in euclidean geometry*) by saying that parallels are lines which do not meet *in any point at a finite distance*! If the teacher of euclidean geometry who has studied projective geometry would read Chapter 6 of this monograph, some of this sort of confusion might be avoided. We find here a clean-cut presentation of the relation between certain ideas of pure projective geometry and such metric concepts as parallel and perpendicular lines; mid-points of line segments and bisectors of angles; types of conics; foci, axes, and directrices of conics; and circles.

Chapter 7 deals with groups of transformations. For one already fairly familiar with the group concept, it is an accurate and lucid presentation. In ten pages one learns of groups, subgroups, transforms of groups, invariant figures under a group, the properties of commutativity and transitivity, and the application of all these ideas to the groups of transformations on one and two-dimensional forms. The reader without considerable mathematical maturity will doubtless find this rather hard going. In discussing the types of point-transformations on a plane, the case with one real and two imaginary fixed points seems to have been left out. One might suppose that it was the intention to include it in Type I if it were not for the statement that "the projectivities on the sides of the invariant triangle are all hyperbolic."

In Chapter 8 there is an introduction of analytic methods based on an algebra of points which is itself based on the group concepts of the preceding chapter. An important fact is given emphasis at this point, namely, that a coordinate system suitable for the study of projective properties may be set up without any use of distance or other metric ideas. The ninth and last chapter presents Klein's concept of a geometry associated with or defined by a group of transformations; and treats very briefly the affine, euclidean metric, and non-euclidean geometries from this point of view.

In the opinion of the reviewer, the book meets a real need in the literature of mathematics in English, and the author has performed a real service in the writing of it.

The Open Court Publishing Company has brought out the book in the very satisfactory size, binding, and typography uniform with the preceding volumes of the series.

W. B. CARVER