A PARADOX OF LEWIS'S STRICT IMPLICATION

BY TANG TSAO-CHEN

The postulates for Lewis's strict implication are nine in number,* namely,

[11.1] \( pq \rightarrow qp \)
[11.2] \( pq \rightarrow p \)
[11.3] \( p \rightarrow pp \)
[11.4] \( (pq)r \rightarrow p(qr) \)
[11.5] \( p \rightarrow \sim (\sim p) \)
[11.6] \( p \rightarrow q.q \rightarrow r: \rightarrow .p \rightarrow r \)
[11.7] \( p.p \rightarrow q: \rightarrow .q \)
[19.01] \( \diamond pq \rightarrow \diamond p \)
[20.01] \( (\exists p, q): \sim (p \rightarrow q). \sim (p \rightarrow \sim q). \)

By the operations of substitution, adjunction, and inference, a body of theorems is obtained. But the following theorem, which is a paradox of the strict implication, is not explicitly mentioned in Lewis's book.

Any two of the first eight postulates are such that each is deducible from the other, if \( p \rightarrow q \) be interpreted as '\( p \) is deducible from \( q \).

In order to prove this theorem we assume the following eight theorems.†

1. \( p \sim p = q \sim q \)
Def. \( 0 = q \sim q \)

* The references are to Symbolic Logic, by Lewis and Langford, 1932.
† For the proof of these theorems see the paper, The theorem "\( p \rightarrow q = p \rightarrow q \) and Huntington's relation between Lewis's strict implication and Boolean algebra, by Tang Tsao-Chen in this Bulletin, vol. 42 (1936), pp. 743-746.
2. \[ p \sim p = 0 \]

3. \[ p0 = 0 \]

\text{Def.} \quad i = \sim \diamond 0

4. \[ pq \sim p. = .i \]

5. \[ p \sim p. = .i \]

6. \[ p \sim q. \sim .i \]

7. \[ p \sim q. = .i . p \sim q \]

8. \[ p \sim q. = .pq = p. \]

Note that the Theorems 4 and 5 are particular cases of the following theorem.

9. \text{If } p \sim q \text{ is asserted, then } p \sim q. = .i.

\begin{align*}
\text{[Hyp.]} & \quad p \sim q & \quad (1) \\
\text{[(1), 8.]} & \quad pq = p & \quad (2) \\
\text{[12.11]} & \quad pq = p. = .pq = p & \quad (3) \\
\text{[(2), (3)]} & \quad pq = p. = .p = p & \quad (4) \\
\text{[11.03, 12.7]} & \quad p = p. = .p \sim p & \quad (5) \\
\text{[(4), (5), 5.]} & \quad pq = p. = .i & \quad (6) \\
\text{[(6), 8.]} & \quad p \sim q. = .i & \quad (7)
\end{align*}

From the above theorem it is very easy to prove the following theorem.

10. \text{If } p \sim q \text{ and } r \sim s \text{ are both asserted, then}

\[ p \sim q. \sim .r \sim s \]

\text{and}

\[ r \sim s. \sim .p \sim q. \]

\begin{align*}
\text{[Hyp.]} & \quad p \sim q & \quad (1) \\
\text{[(3), 9.]} & \quad p \sim q. = .i & \quad (2) \\
\text{[Hyp.]} & \quad r \sim s & \quad (3)
\end{align*}
[5), 9.] \( r \rightarrow s. = i \)  
[(4), (6)] \( p \rightarrow q. = r \rightarrow s \)  
[11.03] \( (7) = (1)(2) \)  
[(7), (8)] \( (1)(2) \)  
[11.2] \( (1)(2) \rightarrow (1) \)  
[12.17] \( (1)(2) \rightarrow (2) \)  
[(9), (10)] \( (1) \)  
[(9), (11)] \( (2) \).

The paradox stated above is a particular case of Theorem 10, and therefore requires no further proof.

NATIONAL WU-HAN UNIVERSITY, 
WUCHANG, CHINA

THE BETTI NUMBERS OF CYCLIC PRODUCTS

BY R. J. WALKER

1. Introduction. In a recent paper† M. Richardson has discussed the symmetric product of a simplicial complex and has obtained explicit formulas for the Betti numbers of the two- and three-fold products. Acting on a suggestion of Lefschetz, we define a more general type of topological product and apply Richardson’s methods to compute the Betti numbers of a certain one of these, the “cyclic” product.

2. Basis for \( m \)-Cycles of General Products. Let \( S \) be a topological space and \( G \) a group of permutations on the numbers 1, \( \cdots \), \( n \). The \textit{product of \( S \) with respect to \( G \),} \( G(S) \), is the set of all \( n \)-tuples \( (P_1, \cdots, P_n) \) of points of \( S \), where \( (P_{i_1}, \cdots, P_{i_n}) \) is to be regarded as identical with \( (P_1, \cdots, P_n) \) if and only if the permutation \( (i_1; \cdots; i_n) \) is an element of \( G \). A neighborhood of \( (P_1, \cdots, P_n) \) is the set of all points \( (Q_1, \cdots, Q_n) \) for which \( Q_i \) belongs to a fixed neighborhood of \( P_i \). It is not difficult to verify that the

† M. Richardson, \textit{On the homology characters of symmetric products}, Duke Mathematical Journal, vol. 1 (1935), pp. 50–69. We shall refer to this paper as R.