by the assumption that an algebraic system of partial differential equations which, corresponding to each unknown, contains at most one equation with a derivative of that unknown for leader (a so-called *normal* system) has a unique solution for each set of initial determinations.

Chapters V and VI, on which later chapters are modeled, are devoted to the study of solutions of systems of polynomial equations and inequations in variables $y_1, \cdots, y_n$. The treatment is simplified to a surprising extent by the admission of the inequation on equal footing with the equation, and it leads to a decomposition of every system into a finite number of canonical systems without common roots. A canonical system embraces a sequence of at most n polynomials, each of which contains variables y_k not occurring effectively in the preceding ones, and its solvability is almost trivial. But it should be pointed out that the author does not try out his treatment on more elaborate questions concerning multiplicity of roots and ideals of polynomials. In Chapters VII and VIII the author presents his decomposition of algebraic systems of partial differential equations into *passive standard* systems. These are systems which, in addition to being canonical (as in Chapter VI), satisfy a condition of "integrability"; and their solution can in turn be made to depend on the successive solution of a finite number of normal systems. The treatment of normal systems in the author's version of Riquier's existence theorem is completed in Chapter X. But the book does not include topics of the type of Ritt's extension of Hilbert's "Nullstellensatz" to differential systems.

In Chapter IX the investigations of the previous chapters are applied to a study, with generalizations of some results to non-linear forms, of Cartan's existence theorems for pfaffian systems. For instance, the integral varieties of a pfaffian system satisfy a related system of partial differential equations, and the solutions of its principal canonical factors are all those *non-singular* integral varieties whose dimension is equal to the genus of the pfaffian system. Finally, Chapter XI gives several examples to illustrate reduction of pfaffian forms, Riquier's dissection of a Taylor series corresponding to a system of monomials, and reduction of polynomial and differential systems.

SALOMON BOCHNER

Integralgleichungen. By G. Hamel. Berlin, Springer, 1937. 8+166 pp.

This book has grown out of a series of lectures delivered in the spring of 1937 at the Extension Institute of the Technische Hochschule in Berlin. The lectures are addressed to men in practical work, with the general purpose of presenting topics not well known to them and of indicating applications of the theory discussed. The success of the lectures on integral equations suggested the desirability of publishing them for the benefit particularly of engineers and physicists.

The book contains no new results of interest to the mathematician and could be used as a textbook only when supplemented by references to original sources and other standard works. The author has, however, achieved considerable success in presenting the fundamental concepts and lines of argument against a background of ideas based upon definite physical problems.

The first part of the book (91 pages) contains the material as presented in the lectures. The different standard types of integral equations are introduced by the familiar problems associated with a vibrating string, and their connection with and solution by differential equations are explained. Emphasis is then confined to linear equations with symmetric kernel. Solution by Neumann's method is given and the integral equations arising from potential theory are introduced, after which the general theory of Schmidt is presented. The theory is summarized in ten fundamental theorems, five of which are stated without proof. The proofs which are given are sometimes intentionally lacking in rigor, in order to achieve brevity for a class of readers more interested in applications than in mathematical logic. Where such gaps have been allowed, the author gives references to other sources.

The second part follows less exactly the original lectures. It is concerned chiefly with the method of Fredholm and includes a brief mention of singular kernels and non-linear integral equations.

W. R. LONGLEY

A Treatise on the Analytical Dynamics of Particles and Rigid Bodies with an Introduction to the Problem of Three Bodies. By E. T. Whittaker. 4th edition. Cambridge University Press, 1937. 14+456 pp.

This excellent text has been reviewed three times in this Bulletin as follows: E. B. Wilson, vol. 12 (1906), pp. 451–458; G. D. Birkhoff, vol. 26 (1920), p. 183; W. R. Longley, vol. 34 (1928), p. 671.

The present edition is identical with the third except that a few errors have been corrected and the references have been brought more nearly up to date.

The account of the problem of three bodies is indeed brief and will be difficult reading for one not already acquainted with the subject. One wonders whether the author has read some recent papers on the problem of three bodies which have appeared in American journals, particularly those which discuss the stability of the equilateral triangle positions for three finite masses. In this connection it may be remarked that the references to recent American papers are incomplete, but this does not detract from the merit of the text, which this reviewer regards as the best in its field in the English language.

H. E. BUCHANAN

Teoria Dinamica dei Regimi Fluidi Turbolenti. By G. D. Mattioli. Padua, Milani, 1937. 323 pp.

In this work, which consists largely of a development of ideas put forward by Mattioli in fifteen papers published in Italian journals and in the Comptes Rendus, the leading idea is that a turbulent mass of fluid consists more or less of discrete elements which mix chaotically, an element being regarded as having both linear momentum and angular momentum about its center of mass. Such a gyrostatic element is supposed to have an ephemeral existence, and its momentary separation from the main body of fluid leads to a local state of instability of flow.

With the aid of these gyrostatic elements of finite size and notions that are generally accepted, equations are set up and then a limiting process is used which Karman thinks is not quite clear, as it seems to eliminate the finiteness of size on which the angular momentum depends. Mattioli has, however, recently written to Karman, explaining his views more fully. In the second part of the book the equations are applied in an able manner to the standard problems of turbulence such as jets, flow through a straight pipe, and flow round a curved channel. In the last case a comparison is made between the theoretical results and some measurements made at Pasadena by Wattendorf.

Among the results of mathematical interest may be mentioned the occurrence of special functions such as the incomplete gamma function, the dilogarithm, and a function defined by an indefinite integral with respect to t in which $Ce^{at}-t$ appears in the denominator.

HARRY BATEMAN