Einführung in die mathematische Logik und in die Methodologie der Mathematik. By Alfred Tarski. Vienna, Springer, 1937. 10+166 pp.

This book is an introduction in the strictest sense; the reader is led just inside one field after another, given a glimpse of interesting things in the interior, and dragged away. It is thoroughly elementary, written for a layman who cannot even be depended upon to know that "*w. z. b. w.*" means "*was zu beweisen war.*" But it is a rare combination of elementariness and competence, and experts can profit from the clear and correct formulation of elementary concepts which the book provides. Particularly noteworthy is the author's care in avoiding the common confusions between sign and object, or between use and mention of expressions.

The book begins with an explanation of the variable, a discussion of definition, and an introduction to the truth functions. Various principles of the truth-function calculus are made clear without recourse to truth tables or even symbolic notation. Then the basic principles of the theory of identity are explained; also the elementary concepts of class theory, stopping short of types; also the principal classifications of relations. Special attention is accorded to functions, which are explained as one-many relations. The method of defining arithmetical concepts in terms of logic is then roughly sketched. Part I concludes with an explanation of the methodological concepts of axiom, formal system, model, consistency, completeness, and independence. The latter three concepts are explained only in their traditional forms (as opposed to Post's formulations), so that they apply only to systems which presuppose elementary logic.

In Part II, three axiom systems for part of arithmetic are compared with a view to illustrating mathematical methodology The book closes with two comprehensive systems for real-number theory. In the course of these developments the reader becomes acquainted with such concepts as group, field, closed system, compactness, and continuity. The excellent exposition of the book is supplemented with an abundance of skillfully devised exercises.

It is only in criticizing this book by its own high standards that a few imperfections of formulation are to be found. Tarski distinguishes sharply between terms (*Bezeichnungen*) and statements (*Sätze*), and explains that whereas the variables "x," "y," $\cdots$, supplant terms, the variables "p," "q," $\cdots$, supplant statements. This is excellent, and calculated to eliminate questions which arise from the confused use of "p," "q," $\cdots$, as nouns: the question "What sort of objects do the variables 'p,' 'q,' $\cdots$ *denote*, or take as *values*?" or the question "What conditions are necessary in order that $p=q$?" But at a few points Tarski himself slips back into the use of "p" and "q" as nouns; thus we read "*Für beliebige p und q* $\cdots$," and again "*Wenn q aus p folgt und* $\cdots$."

Further, it would have been more in the spirit of his careful distinctions to have suppressed the common but confusing terminology "*Implikation*" and "*Equivalenz*" in favor of "conditional" ("*Bedingungssatz*") and "biconditional" ("*gegenseitiger Bedingungssatz*"). The conditional and biconditional are most naturally construed as modes of statement-composition ("if-then," "if and only if") coördinate with alternation ("or") and conjunction ("and"), whereas implication and equivalence are more naturally construed as metalogical relations between statements, expressed by inserting verbs ("implies," "is equivalent to") between names of the statements.

The reader should be warned, finally, against identifying Tarski's *Satzfunktion* with the propositional function of *Principia Mathematica.* The former is not a function at all, in the mathematical sense of a one-many relation, but is rather a *statement form* or *matrix*, an expression derived from a statement by putting variables for constants. In *Principia* the propositional function vacillates vaguely between this status and the status of a property or relation "in intension," the latter status being the one relevant to the formal developments; but Tarski follows the current trend of dispensing with propositional functions in the latter sense in favor of ordinary classes and relations.

W. V. QUINE

Projektive Geometrie. By Karl Doehlemann. New one-volume edition by Heinrich Timerding. (Sammlung Göschen, no. 72.) Berlin and Leipzig, de Gruyter, 1937. 131 pp.

Earlier editions of this title in the Sammlung Göschen consisted of two parts, nos. 72 and 876, written by Karl Doehlemann. One familiar with them will find in the volume under review a new book, entirely rewritten and differing from its predecessors in size, content, and style. It goes by the same title, however, and bears the same number in the collection as Part I of former editions.

For example, the third edition of no. 72, published in 1912, contains 179 pages and an index, and is divided into seven sections of which the first six carry the reader through the pole-polar theory of conics, that is, through the elements of projective geometry, customarily so-called. The seventh section is devoted to cones and ruled surfaces of the second order.

The present volume, on the other hand, attempts a more ambitious program. This will be sufficiently indicated by a list of its chapters: 1. Projektive Grundgebilde in der Ebene; 2. Kurven zweiter Ordnung; 3. Projektive Geometrie des Raumes; 4. Flächen zweiter Ordnung; 5. Raumkurven dritter Ordnung; 6. Kollineationen und Korrelationen. There is no index, but the Inhaltsverzeichnis, in a volume of this size, seems to be sufficient as a guide to particular items.

The first two chapters (59 pages) are devoted to the projective geometry of the plane, swiftly developed in the synthetic manner with considerable appeal to intuition. The double ratio is introduced immediately after central projection and ideal elements (in the plane); and the projective relationship is defined in terms of equal double ratios. Then, after harmonic points have been defined as four points whose double ratio is -1 (when they are taken in a certain order), Desargues' Theorem for two coplanar triangles is given, and is followed by brief discussions of involutions, duality, and the projective ordering of points on a line. With this introduction, the theory of conics, Pascal's Theorem, and poles and polars are developed. Thus, after a short interlude on imaginary elements, the first and second chapters dispose of the material usually considered in an elementary course on projective geometry.

This more or less detailed examination of the two first chapters indicates the nature of the whole treatment. Everything is directed toward one end, to cover ground with the strictest economy of expression. Consequently, the arrangement of topics is found to be somewhat different from the usual arrangement; and the principle of duality is seldom invoked after its initial statement. We find, for example, Brianchon's Theorem, in the second chapter, discussed only as an illustration of the theory of polar reciprocals; and, in the fourth chapter, ruled surfaces are introduced as particular types of surfaces of the second order.

Whatever the disadvantages of such a rapid flight over the field (and I am inclined to feel that much of the essence of geometry is missed by such an enumeration of its facts only, without mentioning its logical implications), it does take one to unfrequented places. Thus I would note the fifth chapter, on space curves of third order,