bounding surface S by F, and the integral of the mean curvature of S extended over S by M. Let K_1 be a movable convex body, with corresponding invariants V_1, F_1, M_1. Then the measure of the set of all positions of K_1 cutting K is given by $8\pi^2(V+V_1)+2\pi(M_1F+MF_1)$. The following is an illustration of the manner in which such a result applies to the theory of geometric probability. Let $\overline{K}$ be a convex body inside K. Then the probability that a third convex body K_1 which cuts K also cuts $\overline{K}$ is

$$W = \frac{4\pi(\overline{V} + V_1) + (M_1\overline{F} + \overline{M}F_1)}{4\pi(V + V_1) + (M_1F + MF_1)}.$$

The third pamphlet contains a study of point, straight line, and kinematic measures in the euclidean plane, with applications. Known formulas of Crofton, Poincaré, and Minkowski are proved anew. Some results are proved first for closed convex regions, and later extended to general complexes. An excellent and complete (up to 1936) bibliography is given at the end.

The fourth pamphlet is a continuation of the third, and obtains analogous results for the invariant measures in 3-space, both for closed convex regions and regions bounded by polyhedra. The bibliography is brought up to date.

The last pamphlet is a historical survey of a question in differential geometry in the large, which may be considered as arising from a "theorem" of Euclid. Euclid states that two polyhedra in space are congruent if the faces of one are congruent to the faces of the other. Since any polyhedron can be obtained by identifying pairs of sides in some plane polygonal net, two questions naturally come up: (1) Corresponding to a given plane polygonal net is there always a polyhedron in 3-space? (2) Is this polyhedron unique (that is, are any two such polyhedra congruent under euclidean notions)? Neither of these questions has an affirmative answer without qualification. Cauchy and others have given proofs of the uniqueness theorem (2) under the added hypothesis of convexity of the polyhedron. The existence theorem (1) is not true even under hypotheses on the polygonal net sufficient to insure the closure, orientability, and simply-connectedness of any resulting polyhedron, as well as certain natural metric assumptions on the net. In 1915 Weyl generalized the formulation of these questions as follows: (1) Corresponding to a given two-dimensional Riemannian metric with positive curvature is there always an ovaloid (closed convex surface) in E_3? (2) Is this ovaloid isometrically unique? The uniqueness theorem has been proved by several authors. A proof is given here which holds for polyhedra as well as surfaces with continuous curvature. A proof of the existence theorem has been sketched by Weyl and completed by Lewy. A new possible method of proof, based on the calculus of variations, is sketched by Blaschke.

All five of these pamphlets are well written, and should be of interest both to geometers and students of probability.

S. B. MYERS

Theory of the Integral. By Stanislaw Saks. 2d revised edition. English translation by L. C. Young. Monografje Matematyczne, vol. 7. Warsaw, 1937. 6+347 pp.

This is the third edition of the excellent and eminently useful book by Saks (the first appeared in 1930, in Polish, and the second in 1933, in French; the latter was reviewed in this Bulletin, vol. 40 (1934), pp. 16–18). It is, however, almost a new book, due to numerous changes in exposition and order of the material and important additions of new topics treated. The opening chapter, I (The integral in an abstract space), treats of the modern theory of abstract measure and integration. The basis is specialized in Chapter II (Carathéodory measure) and Chapter III (Functions of bounded variation and the Lebesgue-Stieltjes integral). Chapter IV (Derivation of additive functions of a set and of an interval) contains a considerable amount of new material, in particular an exposition of important investigations of Ward. It is followed by Chapter V (Area of a surface $z=f(x, y)$), and Chapter VI (Major and minor functions) which contains an elegant treatment of the Perron integral and applications to the theory of functions of a complex variable (Looman-Menchoff theorem). Results of Chapter VII (Functions of generalized bounded variation) are used in the subsequent Chapter VIII (Denjoy integrals). The last chapter, IX (Derivates of functions of one or two real variables), contains a thorough exposition of results of Banach, Besicovitch, Denjoy, Khintchine, Ward, and many other authors. The book closes with two appendices by Banach (On Haar's measure, and The Lebesgue integral in abstract spaces) and with a ten page list of references. The excellent qualities of the book were sufficiently pointed out in the review of the French edition; they explain the remarkable success of the book. The reviewer has no doubt that a fourth edition, still further improved and augmented, will appear before long.

J. D. TAMARKIN

Modern Theories of Integration. By H. Kestelman. Oxford, Clarendon Press, 1937. 8+252 pp.

"The book is intended primarily for students who have covered the ground of G. H. Hardy's *Pure Mathematics* · · · . The account of the Lebesgue integral which is developed in this book follows the lines of C. Carathéodory's *Vorlesungen über reelle Funktionen* · · · ." The following list of contents gives an adequate idea of the material contained in the book; the numerals are chapter numbers: I, Sets of points; II, Riemann integration; III, Lebesgue measure; IV, Sets of ordinates and measurable functions; V, Lebesgue integral of a non-negative function; VI, Lebesgue integrals of functions which are sometimes negative; VII, Functions of a single variable; VIII, Evaluation of double integrals; IX, Extensions of the Lebesgue integral; X, Fourier series. Chapter IX contains a simplified treatment of the Denjoy integral due to Romanovski. The exposition is rather detailed, but the title of the book appears to the reviewer somewhat misleading inasmuch as Stieltjes integrals and integration in abstract spaces are not even mentioned.

J. D. TAMARKIN