The "principle of tolerance" is explicitly restricted to uninterpreted logistic calculi and it is said that "a system of logic is not a matter of choice, but either right or wrong, if an interpretation of the logical signs is given in advance." The quoted sentence—by failing to take account of the fact that an interpretation, in advance of *some* formalization, must have a considerable element of vagueness—may even admit too much to the anti-conventionalists. The author adds, however: "It is important to be aware of the conventional components in the construction of a language system."

The purely syntactical method of the author's previous publications is here supplemented by an account of semantics. Designata are admitted not only for concrete terms but also, in some cases at least, for abstract symbols and expressions. Thus predicates are said to designate properties of things (p. 9), (declarative) sentences are allowed to designate "states of affairs" (p. 11), and "functors" are said to be signs for functions (p. 57). (The more usual terminology is "proposition" instead of "state of affairs" and "function symbol" instead of "functor.") The reviewer would prefer a still more liberal admission of abstract designata, not on any realistic ground, but on the basis that this is the most intelligible and useful way of arranging the matter—it would apparently be meaningless to ask whether abstract terms *really* have designata, but it is rather a matter of taste or convenience whether abstract designata shall be postulated.

The point brought out in §16, that a postulate set in the usual mathematical sense must be regarded as added to an underlying system of logic—which, for exactness, must be logistically formalized—is, of course, not new. But it deserves attention, because neglect of just this point has resulted in much misunderstanding concerning the significance of a set of postulates for a particular mathematical discipline.

On page 23, instead of distinguishing between finite and transfinite rules, it would seem to be better to distinguish between effective and non-effective rules. The matter is complicated by the fact that "finite" is often used in this connection substantially as a synonym of "effective." But a rule might well be non-effective without being transfinite in Carnap's sense.

In §14 there appears to be an oversimplification of the relation between logic and arithmetic, partly through failure to make explicit mention of the axiom of infinity, and partly through an unsound use of recursive definition. An example of the latter is Definition 14, which is in effect a schema providing separate definitions for $m+0$, $m+1$, $m+2$, $\cdots$. That this is no definition of the function $+$ may be seen by considering that the sentence, "For all natural numbers x and y, $x+y=y+x$," for example, remains undefined. This section (like most of the monograph) undertakes only to provide an outline statement with omission of formal detail; nevertheless it seems to the reviewer that an unfortunately misleading impression is given.

ALONZO CHURCH

Elements of the Topology of Plane Sets of Points. By M. H. A. Newman. Cambridge, University Press, 1939. 216 pp.

"This book," according to the scholarly description which appears on the jacket, "has the double purpose of providing an introduction to the methods of topology and of making accessible to analysts the simple modern technique for proving the theorems on sets of points required in the theory of functions of a complex variable [separation theorems, for example]."

There is no doubt that for non-topologists at least, many of the proofs of the Jordan separation theorem which have appeared in the literature are either dull or else too rapid for comfort. In the present work, the Jordan theorem and related topics are treated in such a manner that there can be no complaint on either score. The author first introduces a very sensible treatment of complexes, the "simple modern technique" referred to above. Let us imagine a grating formed in the plane by a number of vertical and horizontal lines. Systems of one- and two-dimensional complexes could hardly be simpler than those consisting respectively of the polygonal lines and polygonal regions formed from the grating; methods of combining complexes could hardly be more geometric and intuitive than those defined in terms of modulo two algebra. Moreover, a given grating can be indefinitely "refined" simply by interpolating more lines. The complexes formed from a grating and its refinements constitute a most useful system: it furnishes two-complexes which will approximate any open set with arbitrary closeness and one-complexes which will separate two disjoint closed sets however near together they are.

An explanation of these simple ideas leads at once to the following lemma, due to Alexander. Let Z be the plane closed by the addition of a single ideal point at infinity. Let x and y be points and F_1, F_2 closed sets in Z. On some grating let κ_1 and κ_2 be one-complexes joining x and y and such that κ_i fails to meet F_i, $(i=1, 2)$. Then if x and y are separated by the set F_1+F_2, the polygon $\kappa_1+\kappa_2$ cannot be the boundary of a two-complex in $Z-F_1F_2$. This lemma, the proof of which requires but a few lines, is shown by the author to be one of the sharpest tools in the theory of separation, if skillfully handled. It is used here to settle quickly the decisive points in such questions as the proof of the Jordan theorem, accessibility, the invariance of dimensionality and regionality, the mapping of a simply connected domain and its "boundary elements" onto a closed circular region.

These topics definitely concern sets in the plane, and form the content of Part II. Part I however contains a discussion of sets in general, metric spaces in particular. An account of the fundamental concepts of topology terminates with the definitions of local connectedness and the intrinsic characterizations of simple closed curves and simple arcs. The characterization of the simple closed curves later forms the basis for easy proofs of the so-called converses of the Jordan theorem.

The last chapter contains an account of connectivity theory "in miniature." Only the first two connectivity numbers enter the discussion: p_0 defined for arbitrary plane sets, and p_1 defined for open plane sets. At first sight it would seem that a theory about p_0 and p_1 could hardly be more than trivial. But this is not the case. There is for example a duality theorem of the Alexander type for closed sets in Z—that is, a relation between $p_0(E)$ and $p_1(Z-E)$ where E is a closed set in Z. Moreover, a proof of the invariance of p_1 is not entirely trivial. We believe that here it would have been well to state precisely the meaning of invariance in Theorem 4.1. This theorem asserts bluntly that p_1 is a topological invariant. But one gathers from Part I that a topological invariant of a metric space E is a property that is possessed by every metric space homeomorphic to E. The theorem in question, however, can only mean that for two open plane sets which are homeomorphic, the values of p_1 must be equal. It is possible that the alert student will at this point sense the limitations of the connectivity theory that is presented, and will begin to wonder about suitable definitions for general spaces. Thus he will enter a stream of thought which has passed through fertile regions but which seems as yet not to have reached its ultimate destination.

It seems to the reviewer that the author has accomplished admirably his two-fold purpose. The student will find here an introduction to combinatorial topology which is simple, yet thoroughly scientific. To be sure he will not encounter the delights of the torus and the Möbius strip, but he will see the theory of complexes usefully employed. The analyst will find the necessary separation and mapping theorems treated with skill and clarity. It should be added that he will also find a short readable proof of the Cauchy formula $\int_J f(z) = 0$ in which $f(z)$ is assumed to be regular over the inner domain D of the rectifiable simple closed curve J and continuous in $D+J$. Here again the power of Alexander's lemma shows itself in solving rapidly the separation problems that arise.

P. A. SMITH

Étude Critique de la Notion de Collectif. By Jean Ville. (Monographies des Probabilités, no. 3.) Paris, Gauthier-Villars, 1939. 144 pp.

Professor Ville has written an interesting and valuable discussion of the concept of a collective, upon which many mathematicians found the theory of probability. The author discusses systems of play in detail, and generalizes this idea to that of a "martingale." This leads to a new criterion for the exclusion of sequences from probability discussions, that is, to a new definition of collective. Any given set of sequences of probability 0 can be excluded by this new criterion, whereas the system criterion, used by Copeland, Popper, Reichenbach, Tornier, Wald, can be used only to exclude certain sets of sequences (necessarily of probability 0). Ville extends the definition of a martingale to the case of a stochastic process depending on a continuous parameter, and shows that some of his sequence results go over.

It is unfortunate that this book, which contains much material which clarifies the subject, should contain so much careless writing. This ranges from uniformly incorrect page references to mathematical errors. Thus (p. 46) it is claimed (and used in a proof) that every denumerable set is a G_δ. The author's main theorem on systems is not as strong as earlier results with which he is apparently unfamiliar. (Cf. Z. W. Birnbaum, J. Schreier, Studia Mathematica, vol. 4 (1933), pp. 85–89; J. L. Doob, Annals of Mathematics, (2), vol. 37 (1936), pp. 363–367.) His discussion of random functions is inadequate and obscure, for example, his demonstration that his main theorem on martingales does not go over to the continuous process uses as an example a measure on function space not in accordance with the usual definition of probability measures on this space.

A specialist who can overlook such slips will find many stimulating ideas in this book. Other readers can profit by the comparative analysis of the different criteria for collectives, and by the discussion of martingales.

J. L. DOOB

Problems in Mechanics. By G. B. Karelitz, J. Ormondroyd, and J. M. Garrelts. New York, Macmillan, 1939. 9+271 p.

This is a collection of nearly 800 problems in statics, kinematics and dynamics. Some two thirds are based on those compiled by the late I. V. Mestchersky, of the Polytechnic Institute of St. Petersburg. The authors have not only translated these, but have replaced the metric by English engineering units and given them a background suitable to American students.

The book is intended to supplement a first course in mechanics as applied to engineering. Thus the problems vary from simple exercises in resolution of forces and falling bodies to those on tensions in cables and curvilinear motion under central forces. Many will provide hard practice in the application of mechanical principles, but none are of the puzzle type. Only rudimentary calculus or differential equations and no knowledge of Lagrange's equations is assumed. As is the case in actual engineering practice, with few exceptions the problems are reducible to those in one or