The nature of the constant $A_4$ here remains undetermined just as in the papers of Rutledge and Douglass. Whether or not it can be rationally expressed in terms of the constants $s_1, \sigma_1, s_2$ and $\pi$ is an open question. Some light may be thrown on the problem by a further study of the function $\xi_1(x)$ treated briefly by Nielsen. His definition is as follows,

\begin{equation}
\xi_1(x) = \int_0^1 \frac{\log (1 + t)}{1 + t} t^{x-1} dt, \quad R(x) > 0.
\end{equation}

From this equation and (27) it follows that

\begin{equation}
A_4 = 5s_4/16 - \xi_1^{(3)}(1).
\end{equation}

This in itself, of course, sheds no light but if a relation analogous to (16) could be found involving the function $\xi_1(x)$, it would seem that the question could be answered.

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THE COMPUTATION OF THE SMALLER COEFFICIENTS OF $J(\tau)$

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The purpose of this note is to call attention to the fact that the first twenty-five coefficients $a_0, a_1, \ldots, a_{24}$ in the expansion

\begin{equation}
1728J(\tau) = e^{-2\pi i \tau} + \sum_{n=0}^{\infty} a_n e^{2\pi i n \tau}
\end{equation}

can be computed with relative ease, making use of H. Gupta's tables of the partition function which extend to $n = 600$.

From the multiplicator equation of fifth order of $J(\tau)$ we have

\begin{equation}
1728J(\tau) = y^{-1} + 6 \cdot 5^5 y + 63 \cdot 5^5 y^2 + 52 \cdot 5^8 y^3 + 63 \cdot 5^{10} y^4 + 6 \cdot 5^{13} y^5 + 5^{15} y^6,
\end{equation}

with

* N. Nielsen, loc. cit., p. 233.
† Harrison Research Fellow.
§ Klein-Fricke, Vorlesungen über die Theorie der elliptischen Modulfunktionen, vol. 2, p. 61, formula (11), with the values given in vol. 2, p. 64, (5) and vol. 1, p. 154, (1).
\[ y = 5^{-3} \frac{\Delta(\tau, 1/5)^{1/4}}{\Delta(\tau, 1)^{1/4}} = e^{2\pi i r} \phi(e^{10\pi i r})^6 \phi(e^{2\pi i r})^{-6}, \quad \phi(x) = \prod_{n=1}^{\infty} (1 - x^n). \]

On the other hand we have

\[ \sum_{n=0}^{\infty} \rho(25n + 24)e^{2\pi i n r} \]

\[ = \phi(e^{2\pi i r})^{-1} e^{2\pi i r} \left\{ 63 \cdot 5^2 y + 52 \cdot 5^3 y^2 + 63 \cdot 5^5 y^3 + 6 \cdot 5^{10} y^4 + 5^{12} y^8 \right\}. \]

Combining (2) and (3) we find

\[ 1728 J(\tau) = y^{-1} + 6 \cdot 5^8 + 5^3 \phi(e^{2\pi i r}) e^{2\pi i r} \sum_{n=0}^{\infty} \rho(25n + 24)e^{2\pi i n r} \]

and hence

\[ x^{-1} + \sum_{n=0}^{\infty} a_n x^n = x^{-1}\phi(x^6)^{-6}\phi(x)^6 + 6 \cdot 5^3 + 5^3 x\phi(x) \sum_{n=0}^{\infty} \rho(25n + 24)x^n. \]

Equation (4) may be used to compute the \( a_n \). The first term of the right member can be expanded with the aid of Jacobi’s formula for \( \phi(x)^6 \) while the expansion of \( \phi(x) \) in the second term is given by Euler’s formula. Thus we may write (4) as

\[ x^{-1} + \sum_{n=0}^{\infty} a_n x^n = \sum_{n=-1}^{\infty} b_n x^n + 5^3(x - x^2 - x^3 + x^5 + x^8 - x^{13} - x^{16} + x^{23} + x^{27} - \cdots) \sum_{n=0}^{\infty} \rho(25n + 24)x^n, \]

with the values

\[
\begin{align*}
  b_{-1} &= 1, & b_0 &= 744, & b_1 &= 9, & b_2 &= 10, \\
b_3 &= -30, & b_4 &= 6, & b_5 &= -25, & b_6 &= 96, \\
b_7 &= 60, & b_8 &= -250, & b_9 &= 45, & b_{10} &= -150, \\
b_{11} &= 544, & b_{12} &= 360, & b_{13} &= -1230, & b_{14} &= 184, \\
b_{15} &= -675, & b_{16} &= 2310, & b_{17} &= 1410, & b_{18} &= -4830, \\
b_{19} &= 750, & b_{20} &= -2450, & b_{21} &= 8196, & b_{22} &= 4920, \\
b_{23} &= -16180, & b_{24} &= 2376, & & & & & .
\end{align*}
\]

By using (5) and the table of partitions, the values of the $a_n$ may be found by mere additions, subtractions, and a single multiplication by 5.

The following list contains the values of the $a_n$ computed with the aid of Gupta's tables. The values for $n \leq 7$ agree with those given by W. E. H. Berwick.

\begin{align*}
a_0 &= 744, \\
a_1 &= 1 \, 96884, \\
a_2 &= 214 \, 93760, \\
a_3 &= 8642 \, 99970, \\
a_4 &= 2 \, 02458 \, 56256, \\
a_5 &= 33 \, 32026 \, 40600, \\
a_6 &= 425 \, 20233 \, 00096, \\
a_7 &= 4465 \, 69940 \, 71935, \\
a_8 &= 40149 \, 08866 \, 56000, \\
a_9 &= 3 \, 17644 \, 02297 \, 84420, \\
a_{10} &= 22 \, 56739 \, 33095 \, 93600, \\
a_{11} &= 146 \, 21191 \, 14995 \, 19294, \\
a_{12} &= 874 \, 31371 \, 96857 \, 75360, \\
a_{13} &= 4872 \, 01011 \, 7981 \, 42520, \\
a_{14} &= 25497 \, 82738 \, 94105 \, 25184, \\
a_{15} &= 1 \, 26142 \, 91646 \, 57818 \, 43075, \\
a_{16} &= 5 \, 93121 \, 77242 \, 14450 \, 58560, \\
a_{17} &= 26 \, 62842 \, 41315 \, 07752 \, 45160, \\
a_{18} &= 114 \, 59912 \, 78844 \, 47865 \, 13920, \\
a_{19} &= 474 \, 38786 \, 80123 \, 41688 \, 13250, \\
a_{20} &= 1894 \, 49976 \, 24889 \, 33900 \, 28800, \\
a_{21} &= 7318 \, 11377 \, 31813 \, 75192 \, 45696, \\
a_{22} &= 27406 \, 30712 \, 51362 \, 46549 \, 29920, \\
a_{23} &= 99710 \, 41659 \, 93718 \, 26935 \, 33820, \\
a_{24} &= 3 \, 53074 \, 53186 \, 56142 \, 70998 \, 77376.
\end{align*}

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