
The purpose of this book, according to the author, is "to present the most elementary course possible on this subject" and at the same time "to emphasize those notions which seem to be proper to linear spaces." Despite the assertion that these aims are not antagonistic, the exposition would be pretty tough going for the average graduate student. Although the reader is not assumed, except in an isolated section, to know about Lebesgue integration, and although the proof of such a comparatively elementary fact as that a continuous image of a compact set is compact is given in detail (p. 48), many parts of the book assume a great deal more sophistication.

The discussion is almost entirely unmotivated: the beginner might like to know why one studies spectral families, or the adjoints of operators. Even to one familiar with the theory it requires proof that von Neumann's definition of $T^*$ is equivalent to the easier one usually given for bounded transformations; $T^*$ is defined as the negative of the transformation whose graph is the orthogonal complement of the graph of $T$.

Concerning the author's choice of the order of the material, it is questionable whether or not it is pedagogically advisable to aim the
discussion at unbounded operators from the very beginning, particu­
larly since the main theorem (the spectral representation of self ad­
joint operators) is first proved for the bounded case. It would seem
better to the reviewer to expound all the easy theory of bounded
operators first, and thus prepared to call attention to the delicate
considerations necessary to study the unbounded case. Also, in Chap­
ter II we find practically the same proof used twice, once to establish
the Riesz theorem on the representation of a bounded linear func­
tional by an inner product (Theorem IV), and once to prove the
possibility of projection on any closed linear manifold in Hilbert
space (Theorem VI). The extremely elementary derivation, due to
Riesz (Acta Szeged, vol. 7 (1934–1935), p. 37), of the former from the
latter, could have been used here to good advantage.

The book contains many minor slips and typographical errors
which may cause serious confusion. Thus in the statement of the
axioms for a linear space (p. 4) the assumption \(1f=f\) is omitted, and
on p. 34, Theorem I, which is stated for an arbitrary transformation,
is proved by reference to a lemma valid only for the additive case.
Regrettable also is the author's reluctance to give to well known the­
orems their usual names: Schwarz's inequality, Bessel's inequality,
Parseval's identity, and the Riesz-Fischer theorem are all indis­
criminately referred to as Theorem \(n\) of Chapter \(m\).

In Chapter VII (footnotes, pp. 67–68) there are some pretty exam­
ples of spectral families, and the Hellinger integral is treated coura­
geously from the modern point of view and not reduced by means of
weak convergence to the classical numerical case. The last two chap­
ters are an excellent idea, carried out unfortunately too rarely: they
contain quick sketches of further developments and applications, and
references to the literature. On the whole the book is a compact and
unified presentation of a well defined part of Hilbert space theory,
and as such may appeal to the reader interested in learning only that
part of the theory which even a non-specialist often needs.

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