described in this brief discussion, it is found that the third integer is
the sum of three biquadrates, and gives Norrie's solution
\[ d = 198593 = \frac{353^4 - 272^4}{15^4} = 21^4 + 8^4 + 2^4. \]

The seventh integer furnishes a second numerical solution
\[ d = 5086913 = \frac{651^4 - 599^4}{10^4} = 43^4 + 34^4 + 24^4. \]

It is the intention of the writer to carry the investigation of each
case as far as values of \( x < 1250 \) \((= 2 \cdot 5^4)\). The fact that an additional
solution was obtained in the first case examined would seem to give
some promise that other solutions exist among the lower integers.

GLENSIDE, PA.

ON THE INVERSION OF THE \(q\)-SERIES ASSOCIATED
WITH JACOBIAN ELLIPTIC FUNCTIONS

A. N. LOWAN, G. BLANCH AND W. HORENSTEIN

The elliptic functions \( sn(u, k) \), \( cn(u, k) \) and \( dn(u, k) \) may be com-
puted from theta functions by well known methods outlined in stand-
ard texts. (See, for instance, Whittaker & Watson, Modern Analysis,
4th edition, p. 485.) Given the modulus \( k \), and \( k' = (1 - k^2)^{1/2} \), there is
associated with \( k, k' \) a function \( \epsilon \) defined by

\[ \epsilon = \frac{1}{2} \left[ \frac{1 - (k')^{1/2}}{1 + (k')^{1/2}} \right] = \frac{1}{2} \frac{\vartheta_2(0, q^4)}{\vartheta_3(0, q^4)}. \]

Values of theta functions for a given parameter \( q \) can be readily
computed, and the Jacobi elliptic functions turn out to be ratios of
the theta functions.

The series for \( \epsilon \) in terms of \( q \) is given by

\[ \epsilon = \sum_{k=0}^{\infty} q^{(2k+1)^2} \sqrt{1 + 2 \sum_{k=1}^{\infty} q^{4k^4}}. \]

1 The above results were obtained in the course of work carried out by the Mathe-
matical Tables Project, conducted by the Work Projects Administration for the City
of New York under the sponsorship of the National Bureau of Standards.
Since, however, the known parameter in elliptic functions is $\epsilon$ and not $q$, the problem of inverting formula (1) arises. Weierstrass$^2$ gave the first four terms of this expansion, and Milne-Thomson$^3$ added two more, giving

$$q = \epsilon + 2\epsilon^5 + 15\epsilon^9 + 150\epsilon^{13} + 1707\epsilon^{17} + 20,910\epsilon^{21} + \cdots.$$  

As is well known, the above series converges for $|\epsilon| < 1/2$; and it is rapidly convergent over a large range of $\epsilon$. But when $\epsilon$ is greater than 0.3, only a few decimals in $q$ may be obtained from the six terms given above. Because of the importance of this series in the computation of elliptic functions as well as in the Jacobian zeta functions, the above series has been extended to fourteen terms, as given below:

$$q = \epsilon [1 + 2u + 15u^2 + 150u^3 + 1707u^4 + 20,910u^5 + 268,616u^6$$
$$+ 3,567,400u^7 + 48,555,069u^8 + 673,458,874u^9$$
$$+ 9,481,557,398u^{10} + 135,119,529,972u^{11}$$
$$+ 1,944,997,539,623u^{12} + 28,235,172,753,886u^{13} \cdots],$$  

where $u = \epsilon^4$.

The following accuracy may be obtained with the above series:

<table>
<thead>
<tr>
<th>$\epsilon$</th>
<th>Accuracy in $q$ with 14 terms of (2)</th>
<th>Accuracy in $q$ with 6 terms of (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.4</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>.36</td>
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<td>5</td>
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<td>.30</td>
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<td>6</td>
</tr>
<tr>
<td>.25</td>
<td>18</td>
<td>8</td>
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