GENERATORS OF PERMUTATION GROUPS SIMPLY ISOMORPHIC WITH \( LF(2, p^n) \)

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It is well known that the group \( LF(2, p^n) \) of linear fractional transformations of determinant unity in the \( GF[p^n] \) can be represented as a permutation group \( G \) of degree \( p^n + 1 \). The purpose of this note is to show that the generators of \( G \) follow from a slight extension of an argument used in a recent paper.¹

We obtain a representation of the abstract group \( L \) simply isomorphic with the special linear homogeneous group \( SLH(2, p^n) \) by means of the cosets \( K \) and \( KTS_\lambda \), where \( \lambda \) ranges over the \( p^n \) marks of the field \( u_0(=0), u_1, \ldots, u_m, (m=p^n-1) \). Let \( k_m=K \) and \( k_u=KTS_u \) for \( i=0, 1, \ldots, m \).

If \( p \) is any mark, \( KS_p=K \) and \( KTS_\lambda \cdot S_p=KTS_{\lambda+p} \), so that to \( S_p \) there corresponds the permutation

\[
(1) \quad s_p = \begin{pmatrix} k_\infty & k_0 & k_{u_1} & \cdots & k_{u_m} \\ k_\infty & k_p & k_{u_1+p} & \cdots & k_{u_m+p} \end{pmatrix}.
\]

If \( \lambda \neq 0 \), \( KTS_\lambda T=KTS_{-\lambda-1} \). Further, \( KTS_0 T=K \), so that to \( T \) there corresponds the permutation

\[
(2) \quad t = (k_0k_\infty \cdot k_{u_1}k_{-u_1-1} \cdots k_{u_m}k_{-u_m-1}).
\]

Hence \( L \) has a \((d, 1)\) isomorphism with \((s_p, t)\), where \( d \) is the order of a subgroup of \( K \) which is invariant in \( L \). The quotient group \((s_p, t)\) is simply isomorphic² with \( LF(2, p^n) \) and is of order \( p^n(p^{2n}-1)/d \), where \( d=2 \) or \( 1 \) according as \( p>2 \) or \( p=2 \).

**Theorem.** A permutation group simply isomorphic with the group \( LF(2, p^n) \) of linear fractional transformations of determinant unity in the \( GF[p^n] \) is generated by \((1)\) and \((2)\), where \( p \) ranges over an independent set of additive generators of the field.

**Corollary.**³ A permutation group simply isomorphic with the group \( LF(2, p) \) is generated by \((k_0k_1k_2 \cdots k_{p-1})\) and \((k_0k_\infty \cdot k_1k_{i_1} \cdot k_2k_{i_2} \cdots)\), where \( ji_j \equiv -1 \) (mod \( p \)).

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¹ A note on the special linear homogeneous group \( SLH(2, p^n) \), this Bulletin, vol. 47 (1941), pp. 629–632. The notation and results of this paper are assumed above.


³ Compare with \( x'=x+1 \) and \( x'=-1/x \), which generate \( LF(2, p) \).