TOPOLOGY


If \( A \) and \( B \) are separable metric spaces, \( B \) nondegenerate, a single-valued continuous transformation \( f(A) = B \) is said to be arc reversing provided the inverse of every simple arc in \( B \) is a simple arc in \( A \). After several basic results are obtained, it is shown that if \( f(A) = B \) is arc reversing, with \( A \) locally compact and \( B \) a locally connected generalized continuum, then the sets \( A - A_1 \) and \( B - B_1 \) are homeomorphic, where \( B_1 \) is the set of all points \( b \) in \( B \) whose inverses \( f^{-1}(b) \) are nondegenerate and \( A_1 = f^{-1}(B_1) \). In particular, if \( f(A) = B \) is (1-1), \( A \) and \( B \) are homeomorphic. A characterization of arc reversing transformations is afforded by the result that a single-valued continuous transformation \( T(A) = B \), where \( A \) and \( B \) are locally connected continua, is arc reversing if and only if the set of inverses \( \{ T^{-1}(b) \} \), \( b \) in \( B \), consists of single points and at most a countable number of free arcs whose end points are of Urysohn-Menger order at most 2 in \( A \). Further results are obtained concerning local separating points and continua having homeomorphs of finite linear measure. (Received December 29, 1943.)

100. Mariano García: Component orbits under pointwise recurrent homeomorphisms.

A point \( x \) of a separable metric space \( X \) on which a homeomorphism \( f(X) = X \) is defined is called recurrent under \( f \) if, given any neighborhood \( U \) of \( x \), there exists an \( N \) such that \( f^n(x) \subseteq U \), and an invariant set \( L \) in \( X \) whose components can be ordered in a sequence \( \cdots, A_1, A_2, A_3, \cdots \) such that \( f(A_i) = A_{i+1} \) is defined as a component orbit. Using methods analogous to those used by Whyburn in proving the results that Hall and Schweigert obtained relative to periodic component orbits (component orbits having a finite number of components) under a pointwise periodic homeomorphism on \( X \), this paper establishes extensions of these results to non-pointwise periodic mappings. It is shown for example that if \( f(X) = X \) is a homeomorphism on a compact space \( X \) and \( G_1, G_2, \cdots \) is a sequence of component orbits whose limit inferior contains a periodic component orbit \( Q \), and if (either) each point of \( \limsup G_i = Q \) is recurrent under both \( f \) and \( f^{-1} \) or each point of \( \sum_{i=1}^{\infty} G_i + \limsup G_i \) is recurrent under \( f \), then \( \limsup G_i \) is a periodic component orbit. (Received December 27, 1943.)


Let \( X \) be a topological space and let \( f(X) = X \) be a homeomorphism. A point \( x \) of \( X \) is said to be recurrent under \( f \) provided that to each neighborhood \( U \) of \( x \) there corresponds a positive integer \( n \) such that \( f^n(x) \subseteq U \). A point \( x \) of \( X \) is said to be almost periodic under \( f \) provided that to each neighborhood \( U \) of \( x \) there corresponds a monotone increasing sequence \( n_1, n_2, \cdots \) of positive integers with the properties that the numbers \( n_{i+1} - n_i \) \((i = 1, 2, \cdots)\) are uniformly bounded and \( f^{n_i}(x) \subseteq U \) \((i = 1, 2, \cdots)\). A subset \( Y \) of \( X \) is said to be minimal under \( f \) provided that \( Y \) is nonvacuous, closed and invariant under \( f \), and furthermore \( Y \) does not contain a proper subset with these properties. The following theorems are established: (1) If \( x \in X \) is recurrent under \( f \), then \( x \) is also recurrent under \( f^n \) for every positive integer \( n \). (2) If \( X \) is minimal under \( f \) but not under \( f^k \), where \( k \) is a nonzero integer, then there exists an integer \( n \gg 1 \), such that \( n \) divides \( |k| \) and \( f^n \) gives a finite minimal-set decomposition of \( X \) which
contains exactly \( n \) elements. These theorems are developed and applied to yield further results, among them the following: If \( X \) is a compact metric space and if \( x \in X \) is almost periodic under \( f \), then \( x \) is also almost periodic under \( f^n \) for every integer \( n \).
(Received December 31, 1943.)

102. D. W. Hall: *On rotation groups of plane continuous curves under pointwise periodic homeomorphisms.*

This note makes use of the work of G. T. Whyburn on light interior transformations and orbit decompositions of certain spaces to obtain a theorem by means of which a certain subset of the orbits of points under a pointwise periodic transformation may be given a linear ordering. This theorem is then used to obtain an accessibility theorem for plane continuous curves similar to one previously published by L. Whyburn (Fund. Math. vol. 28 (1937) p. 127), but having the emphasis in the hypothesis placed upon the type of the transformation rather than upon the order of the rotation group under consideration. (Received December 18, 1943.)