DAVID HILBERT AND HIS MATHEMATICAL WORK

HERMANN WEYL

A great master of mathematics passed away when David Hilbert died in Göttingen on February the 14th, 1943, at the age of eighty-one. In retrospect it seems to us that the era of mathematics upon which he impressed the seal of his spirit and which is now sinking below the horizon achieved a more perfect balance than prevailed before and after, between the mastering of single concrete problems and the formation of general abstract concepts. Hilbert's own work contributed not a little to bringing about this happy equilibrium, and the direction in which we have since proceeded can in many instances be traced back to his impulses. No mathematician of equal stature has risen from our generation.

America owes him much. Many young mathematicians from this country, who later played a considerable role in the development of American mathematics, migrated to Göttingen between 1900 and 1914 to study under Hilbert. But the influence of his problems, his viewpoints, his methods, spread far beyond the circle of those who were directly inspired by his teaching.

Hilbert was singularly free from national and racial prejudices; in all public questions, be they political, social or spiritual, he stood forever on the side of freedom, frequently in isolated opposition against the compact majority of his environment. He kept his head clear and was not afraid to swim against the current, even amidst the violent passions aroused by the first world war that swept so many other scientists off their feet. It was not mere chance that when the Nazis "purged" the German universities in 1933 their hand fell most heavily on the Hilbert school and that Hilbert's most intimate collaborators left Germany either voluntarily or under the pressure of Nazi persecution. He himself was too old, and stayed behind; but the years after 1933 became for him years of ever deepening tragic loneliness.

It was another Germany in which he was born on January 23, 1862, and grew up. Königsberg, the eastern outpost of Prussia, the city of Kant, was his home town. Contrary to the habit of most German students who used to wander from university to university, Hilbert studied at home, and it was in his home university that he climbed the first rungs of the academic ladder, becoming Privatdozent and in due time ausserordentlicher Professor. During his entire life he preserved uncorrupted the characteristic Baltic accent. His reputa-

tion as a leading algebraist was well established when on Felix Klein's initiative he was offered a full professorship at Göttingen in 1895. From then on until the end of his life Hilbert remained in Göttingen. He was retired in 1930.

When one inquires into the dominant influences acting upon Hilbert in his formative years one is puzzled by the peculiarly ambivalent character of his relationship to Kronecker: dependent on him, he rebels against him. Kronecker's work is undoubtedly of paramount importance for Hilbert in his algebraic period. But the old gentleman in Berlin, so it seemed to Hilbert, used his power and authority to stretch mathematics upon the Procrustean bed of arbitrary philosophical principles and to suppress such developments as did not conform: Kronecker insisted that existence theorems should be proved by explicit construction, in terms of integers, while Hilbert was an early champion of Georg Cantor's general set-theoretic ideas. Personal reasons added to the bitter feeling.[1] A late echo of this old feud is the polemic against Brouwer's intuitionism with which the sexagenarian Hilbert opens his first article on "Neubegründung der Mathematik" (1922): Hilbert's slashing blows are aimed at Kronecker's ghost whom he sees rising from his grave. But inescapable ambivalence even here —while he fights him he follows him: reasoning along strictly intuitionistic lines is found necessary by him to safeguard non-intuitionistic mathematics.

More decisive than any other influence for the young Hilbert at Königsberg was his friendship with Adolf Hurwitz and Minkowski. He got his thorough mathematical training less from lectures, teachers or books, than from conversation. "During innumerable walks, at times undertaken day after day," writes Hilbert in his obituary on Hurwitz, "we roamed in these eight years through all the corners of mathematical science, and Hurwitz with his extensive, firmly grounded and well ordered knowledge was for us always the leader." Closer and of a very intimate character was Hilbert's lifelong friendship with Minkowski. The Königsberg circle was broken up when Hurwitz in 1892 left for Zürich, soon to be followed by Minkowski. Hilbert first became Hurwitz's successor in Königsberg and then moved on to Göttingen. The year 1902 saw him and Minkowski reunited in Göttingen where a new chair of mathematics had been created for Minkowski. The two friends became the heroes of the great and brilliant period which our science experienced during the

[1] How Georg Cantor himself in his excitability suffered from Kronecker's opposition is shown by his violent outbursts in letters to Mittag-Leffler; see A. Schoenflies, *Die Krisis in Cantors mathematischem Schaffen*, Acta Math. vol. 50 (1928) pp. 1–23.

following decade in Göttingen, unforgettable to those who lived through it. Klein, for whom mathematical research had ceased to be the central interest, ruled over it as a distant but benevolent god in the clouds. Too soon was this happy constellation dissolved by Minkowski's sudden death in 1909. In a memorial address before the Göttingen Gesellschaft der Wissenschaften, Hilbert spoke thus about his friend: "Our science, which we loved above everything, had brought us together. It appeared to us as a flowering garden. In this garden there are beaten paths where one may look around at leisure and enjoy oneself without effort, especially at the side of a congenial companion. But we also liked to seek out hidden trails and discovered many a novel view, beautiful to behold, so we thought, and when we pointed them out to one another our joy was perfect."

I quote these words not only as testimony of a friendship of rare depth and fecundity that was based on common scientific interest, but also because I seem to hear in them from afar the sweet flute of the Pied Piper that Hilbert was, seducing so many rats to follow him into the deep river of mathematics. If examples are wanted let me tell my own story. I came to Göttingen as a country lad of eighteen, having chosen that university mainly because the director of my high school happened to be a cousin of Hilbert's and had given me a letter of recommendation to him. In the fullness of my innocence and ignorance I made bold to take the course Hilbert had announced for that term, on the notion of number and the quadrature of the circle. Most of it went straight over my head. But the doors of a new world swung open for me, and I had not sat long at Hilbert's feet before the resolution formed itself in my young heart that I must by all means read and study whatever this man had written. And after the first year I went home with Hilbert's *Zahlbericht* under my arm, and during the summer vacation I worked my way through it—without any previous knowledge of elementary number theory or Galois theory. These were the happiest months of my life, whose shine, across years burdened with our common share of doubt and failure, still comforts my soul.

The impact of a scientist on his epoch is not directly proportional to the scientific weight of his research. To be sure, Hilbert's mathematical work is of great depth and universality, and yet his tremendous influence is not accounted for by it alone. Gauss and Riemann, to mention two other Göttingers, are certainly of no lesser stature than Hilbert, but they made little stir among their contemporaries and no "school" of devoted followers formed around them. No doubt this is due in part to the changing conditions of time, but the character of the men is probably more decisive. A taste for solitude, even

obscurity, is in no way irreconcilable with great creative gifts. But Hilbert's was a nature filled with the zest of living, seeking intercourse with other people, above all with younger scientists, and delighting in the exchange of ideas. Just as he had learned from Hurwitz, so he taught his own pupils, at least those in whom he took a deeper personal interest: on far-flung walks through the woods surrounding Göttingen or, on rainy days, as "peripatetics" in his covered garden walk. His optimism, his spiritual passion, his unshakable faith in the supreme value of science, and his firm confidence in the power of reason to find simple and clear answers to simple and clear questions were irresistibly contagious. If Kant through critique and analysis arrived at the principle of the supremacy of practical reason, Hilbert incorporated, as it were, the supremacy of pure reason—sometimes with laughing arrogance (*arrogancia* in the Spanish sense), sometimes with the ingratiating smile of intellect's spoiled child, but most of the time with the seriousness of a man who believes and must believe in what is the essence of his own life. His enthusiasm was compatible with critical acumen, but not with scepticism. The snobbish attitude of pretended indifference, of "merely fooling around with things," or even of playful cynicism, were unknown in his circle. You had better think twice before you uttered a lie or an empty phrase to him: his directness could be something to be afraid of. He was enormously industrious and liked to quote Lichtenberg's saying: "Genius is industry." Yet for all this there was light and laughter around him. He had great suggestive power; it sometimes lifted even mediocre minds high above their natural level to astonishing, though isolated achievements. I do not remember which mathematician once said to him: "You have forced us all to consider important those problems which you considered important." His vision and experience inspired confidence in the fruitfulness of the hints he dropped. He did not hide his light under a bushel. In his papers one encounters not infrequently utterances of pride in a beautiful or unexpected result, and in his legitimate satisfaction he sometimes did not give to his predecessors on whose ideas he built all the credit they deserved. The problems of mathematics are not isolated problems in a vacuum; there pulses in them the life of ideas which realize themselves *in concreto* through our human endeavors in our historical existence, but forming an indissoluble whole transcend any particular science. Hilbert had the power to evoke this life; against it he measured his individual scientific efforts and felt responsible for it in his own sphere. In this sense he was a philosopher, not in the sense of adhering to one of the established epistemological or metaphysical doctrines. Does not in