
ALGEBRA AND THEORY OF NUMBERS


Using the theory of representations of groups the authors have obtained a number of results for groups of certain types of orders. The following result will be proved: If $G$ is a non-cyclic simple group of order $g = pq^b h$, where $p$ and $q$ are primes and where $b$ and $h$ are positive integers with $h < p - 1$, then either $G \cong LF(2, p)$ with $p = 2m + 1 > 3$ or $G \cong LF(2, 2^n)$ with $p = 2^n + 1 > 3$; conversely, these groups satisfy the assumptions. As an application, the authors determine all simple groups of order $pqr^b$, where $p$, $r$, $q$ are primes and where $b$ is a positive integer. The only simple groups of this type are the well known groups of orders 60 and 168. (Received May 23, 1945.)

106. E. T. Browne: Concerning a certain ring of homographies.

In an earlier paper (Ann. of Math. (2) vol. 29 (1928) pp. 483-492) the author considered pairs of $n$-square matrices $A$, $B$ which possess the property that not only $A^2 = B^2 = I$, but also that if $D = (1 - \theta)A + \theta B$, then $D^2 = I$ for every value of the scalar $\theta$. Such a pair $A$, $B$ may be said to determine a linear class of involutions to which $D$ belongs. In this paper for $n$ even he considers certain triples of matrices $A$, $B$, $C$ over the field of complex numbers such that not only $A^2 = B^2 = C^2 = I$, but, in addition, if $D = xA + yB + zC$, then $D^2 = kI$, where $k^2$ depends only on $x$, $y$ and $z$. Call such a triple a symmetrical triple. The four matrices $A$, $B$, $C$, $I$ are shown to be linearly independent and to form the basis of an algebra of order four over the field of complex numbers. Methods are devised for obtaining from a given symmetrical triple other such triples, and in particular triples which are harmonic to the given triple. Geometric interpretations are given for the cases $n = 2$ and $n = 4$. (Received May 25, 1945.)

107. A. P. Hillman: A note on differential polynomials. II.

Let $F \neq 0$ and $G$ be differential polynomials in the unknowns $y_1, \ldots, y_n$. Let $G$ be of order $q_i$ in $y_i$. It is known that if $G$ has all the solutions of $F$, then $G^d = \sum C_i F_i$ where $d$ and $s$ are positive integers, the $C_i$'s are differential polynomials, and $F_i$ is the $j$th derivative of $F$. The author shows that for $j$ greater than the maximum of the $q_i$, $C_i$ is in the perfect ideal generated by $F$ (that is, $C_i$ has all the solutions of $F$). An analogous result holds for partial differential polynomials. (Received May 17, 1945.)

108. L. J. Mordell: Further contributions to the geometry of numbers for non-convex regions.
This gives an application of the methods recently discovered by the author to lattice points in the region \(|x^n + y^n| \leq 1\) where \(m = p/q > 4\) and \(p, q\) are odd integers. (Received April 9, 1945.)

**Analysis**


It is shown that a Tauberian theorem for Nörlund summability given by R. P. Cesco (Universidad Nacional de La Plata, Publicaciones de la Facultad de Ciencias Fisicomatemáticas (2) vol. 4 (1944) pp. 443–445) and more general theorems are implied by familiar Tauberian theorems for Abel summability. (Received May 14, 1945.)

110. R. H. Cameron: *Some examples of Fourier-Wiener transforms of analytic functionals.*

Let \(F(x) = F(x(\cdot))\) be a functional which is defined throughout the space \(K\) of complex continuous functions \(x(t)\) defined on \(0 \leq t \leq 1\) and vanishing at \(t = 0\), and let \(F\) possess the property that \(F(x+iy)\) is Wiener summable in \(x\) over \(C\) for each fixed \(y(\cdot)\) in \(K\). (\(C\) is the subspace of all real functions in \(K\).) Then the functional \(G(y) = \int_C F(x+iy)dx\) is called the Fourier-Wiener transform of \(F(x)\). Examples are given of various functionals \(F(x)\) which have the property that the transforms of their transforms exist and equal \(F(-x)\). The fact that there exist extensive classes of functionals having this property is proved in a paper by W. T. Martin and the author. (Received May 9, 1945.)


Let \(K\) be the space of all continuous complex functions \(x(t)\) defined on \(0 \leq t \leq 1\) which vanish at \(t = 0\), let \(C\) be the subset of all real functions of \(K\), and let \(F(x)\) be a functional defined on \(K\). Then, as defined by one of the authors in another paper, \(G(y) = \int_C F(x+iy)dx\) is called the Fourier-Wiener transform of \(F(x)\) if it exists for all \(y\) in \(K\). The purpose of the present paper is to exhibit three extensive classes of functionals which are taken into themselves in a one-to-one manner by the Fourier-Wiener transformation in such a way that the inverse transformation is \(F(x) = \int_C G(y-ix)dy\). In particular, this property is enjoyed by the class \(E_m\) of functionals \(F(x)\) which are mean continuous, of mean exponential type, and “entire,” that is, continuous in the topology defined by root mean square distance between functions, bounded by \(A \exp \left[ B \int |x(t)^4|dt \right]^{1/4}\), and “entire” in the sense that \(F(x+\lambda y)\) is entire in \(\lambda\) for all \(x\) and \(y\) in \(K\). (Received May 9, 1945.)


The direct product \(E_1 \otimes_N E_2\) of two Banach spaces \(E_1, E_2\) (Trans. Amer. Math. Soc. vol. 53 (1943) pp. 195–217), which naturally depends on the norm \(N\), determines uniquely an “associate space” \(\hat{E}_1 \otimes \hat{N} E_2\) (\(\hat{E}_1\) is the norm “associate” with \(N\)) and a conjugate space \(\overline{E}_1 \otimes_N E_2\). It is shown that for a “natural crossnorm,” \(\overline{L} \otimes \overline{N}\) is a proper subset of \(\overline{L} \otimes \overline{N}\). Similarly, for a “natural crossnorm,” \(\hat{L} \otimes \hat{N}\) is a proper subset of \(\hat{L} \otimes \hat{N}\). A related example of a non-reflexive crossspace \(E_3 \otimes_N E_2\) is constructed, for which \(E_3, E_2,\) and \(N\) are all reflexive. (Received April 23, 1945.)

113. Herbert Federer: *Coincidence functions and their integrals.*