dendron with respect to its elements, and (3) if $g$ and $h$ are elements of $G$ and $H$ respectively, the common part of $g$ and $h$ exists and is totally disconnected. Then $W$ contains a point at which $G$ is hereditarily non-equicontinuous.

Proof. Obtain $g_\alpha, AB, C, \rho,$ and $g$ as in Theorem 7. Of every countable sequence of different elements of $G$ having a subset of $g$ as a limiting set, all but a finite number separate $g$ from $g_\alpha$. Hence $G$ is hereditarily non-equicontinuous at $C$.

References


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DIMENSIONAL TYPES

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Let $H$ and $S$ be topological spaces. We say that $H$ is of dimensional type $S$ (symbol: $D_S$) if for each closed set $X$ and mapping $f:X \to S$ there exists an extension $\tilde{f}:H \to S$.

It is clear that (from a result due to Hurewicz [1, p. 83]) when $H$ is separable metric and $S$ is an $n$-sphere, then $H$ can be of dimensional type $S$ if and only if $\dim H \leq n$. For simplicity we write $D_n$ for $D_S$ when $S$ is an $n$-sphere. It is, of course, possible to define $\dim H$ as the least integer $n$ for which $H$ is of type $D_n$ even when $H$ is not separable metric. But this seems to be open to objection except in certain cases (cf. (d) below).

It is at once clear that we have:

(a) If $H$ is of type $D_S$ then so also is any closed subset.

(b) If the closed sets $H_1$ and $H_2$ are of type $D_S$ then so also is the set $H_1 + H_2$.

As a matter of notation we may suppose that $H = H_1 + H_2$. Let $f:X \to S$. Several cases may arise of which we shall consider only the

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one in which \(X \cdot H_1 \cdot H_2\) is not empty. By (a) the mapping \(g = f|X \cdot H_1 \cdot H_2\) may be extended to a mapping \(\tilde{g}: H_1 \cdot H_2 \rightarrow S\). Let \(g_i = \tilde{g}\) on \(H_1 \cdot H_2\) and \(g_i = f\) on \(X \cdot H_i\), \(i = 1, 2\). Then \(g_i: (H_1 \cdot H_2 + X \cdot H_i) \rightarrow S\) and so by assumption may be extended to a mapping \(\tilde{f}_i: H_i \rightarrow S\). Since \(\tilde{f}_1 = \tilde{f}_2\) on \(H_1 \cdot H_2\) we may combine these mappings into a transformation \(\tilde{f}\) carrying \(H\) into \(S\).

With the aid of (b) and additional hypotheses we can extend (b) to (c). Let \(H\) be a normal space and \(S\) an absolute neighborhood retract. If \(H = H_1 + H_2 + \cdots\) where each \(H_i\) is closed and of dimensional type \(S\) then \(H\) is of dimensional type \(S\).

From (b) there is no loss of generality in supposing that \(H_1 \subset H_2 \subset H_3 \cdots\). Let \(f\) map the closed set \(X\) into \(S\). We may assume that \(X\) meets \(H_1\). Let \(f = f_1\) so that (since \(S\) is an ANR) there is an extension \(\tilde{f}_1\) of \(f_1\) mapping a neighborhood \(U_1\) of \(X\) into \(S\). From the normality of \(H\) it follows that there exists a neighborhood \(V_1\) of \(X\) whose closure is contained in \(U_1\). Let \(X_1 = V_1\) and set \(g_1 = f_1|H_1 \cdot X_1\). By assumption \(g_1\) admits an extension \(\tilde{g}_1: H_1 \rightarrow S\). Let \(f_2 = \tilde{g}_1\) on \(H_1\) and \(f_2 = \tilde{f}_1\) on \(X_1\) so that \(f_2\) is a mapping of \(H_1 + X_1\) into \(S\). We may extend \(f_2\) to \(\tilde{f}_2: U_2 \rightarrow S\), where \(U_2\) is a neighborhood of \(H_1 + X_1\). Let \(V_2\) be neighborhood of \(H_1 + X_1\) for which \(X_2 = V_1 \subset U_2\). Suppose that \(n\) exceeds 2 and assume that \(\tilde{f}_n: U_n \rightarrow S\) is an extension of \(f_r: (H_{n-1} + V_{n-1}) \rightarrow S\) and that \(X_n = V_n \subset U_n\) where \(V_n\) is a neighborhood of \(H_{n-1} + V_{n-1}\). Let \(g_n = \tilde{f}_n|H_n \cdot X_n\) and then denote by \(\tilde{g}_n\) an extension of \(g_n\) to \(H_n\).

Define \(f_{n+1}\) as \(\tilde{g}_n\) on \(H_n\) and put \(f_{n+1} = \tilde{f}_n|X_n\) so that \(f_{n+1}\) maps \(H_n + X_n\) into \(S\). Extend \(f_{n+1}\) to a transformation \(\tilde{f}_{n+1}\) of \(U_{n+1}\) into \(S\) where \(U_{n+1}\) is an open set containing \(H_n + X_n\). Let \(V_{n+1}\) be a neighborhood of this set with \(X_{n+1} = V_{n+1} \subset U_n\).

It follows that:

(i) \(V_1 \subset V_2 \subset V_3 \subset \cdots\)

(ii) \(\tilde{f}_{n+1}\) is an extension of \(\tilde{f}_n\) defined on \(V_{n+1}\), \(\tilde{f}_1\) being an extension of \(f\).

(iii) \(H = \sum_{n=1}^{\infty} V_n\).

If \(x \in H\) then, for some \(n\), \(x \in V_n\) and we may write \(\tilde{f}(x) = \tilde{f}_n(x) = \tilde{f}_{n+1}(x) = \cdots\). That \(\tilde{f}\) is continuous follows from the fact that the sets \(V_n\) are open.

Other generalizations of the classical sum-theorem have been given by P. Alexandroff, E. Čech, and other mathematicians.

To validate the definition suggested earlier it seems necessary to restrict the class of spaces to which it is intended to apply. A space \(H\) will be said to have property \(V\) provided that for any two closed sets \(X_1, X_2\) there exist closed sets \(H_1, H_2\) such that \(H = H_1 + H_2\), \(H_1 \cdot H_2 \cdot (X_1 + X_2) = X_1 \cdot X_2\) and \(X_i \subset H_i\). This is a well known property.
of metric spaces but we have no reference to its formulation in the literature as an axiom.

(d) *If H is of type Dₙ and has property V then H is of type Dₙ₊₁.*

Following a line of argument used by Hurewicz \[2, \text{p. 144}\] let \(f\) map the closed set \(X\) into \(S_{n+1}\). We may assume that \(S_{n+1}\) is given by the equation \(y₁^2 + \cdots + y_{n+3}^2 = 1\) and let \(A₁, A₂\) denote the subsets of \(S_{n+1}\) given by \((y_{n+2} ≥ 0), (y_{n+2} ≤ 0)\). Then \(A₁\) and \(A₂\) are \((n+1)\)-cells which meet in the \(S_n:\(y₁^2 + \cdots + y_{n+1}^2 = 1\). (y_{n+2} = 0)\). We may clearly suppose that \(f(X)\) meets both \(A₁\) and \(A₂\) so that \(X_i = f^{-1}(A_i, f(X))\) is not empty. Since \(H\) has property \(V\) we have \(H = H₁ + H₂\) where \(X_i ⊂ H_i = H_i\), and \(H₁ \cdot H₂ \cdot X = X₁ \cdot X₂\). We consider only the case where this latter set is not vacuous. Denote by \(g\) the mapping \(f\) restricted to \(X₁ \cdot X₂\) so that \(g\) has values lying in \(S_n\). By (a), \(H₁ \cdot H₂\) is of type \(Dₙ\) and so we may extend \(g\) to a mapping \(g: H₁ \cdot H₂ \rightarrow Sₙ\). Then (by Tietze's extension theorem) we may extend \(g\) to a mapping \(gᵢ\) of \(Hᵢ\) into \(Aᵢ\). Since \(g₁ = g₂\) on \(H₁ \cdot H₂\) we combine these mappings to secure an extension \(\tilde{f}\) of \(f\) taking \(H\) into \(S_{n+1}\).

**References**