tion law. (2) For each $n \geq 2$ there is an $n$-sided convex polygon of maximal length inscribed in such a curve. (Received August 9, 1943.)


An earlier abstract (50-5-132) announced a proof without use of fixed point theorems of the following result. If $C$ is a symmetric closed convex curve, there exists a parallelogram $P$ circumscribed about $C$ so that the midpoint of each side of $C$ is on $P$. Using very elementary minimal area methods this result is extended to polygons of an arbitrary number of sides, and to polyhedra about convex bodies in Euclidean space of any finite dimension. In higher dimensions the phrase “midpoint of each side” is replaced by “centroid of each face.” (Received August 9, 1945.)

238. H. P. Pettit: *The tangents at certain multiple points on a curve $C_{2mn}$.*

In a paper published in the Tôhoku Mathematical Journal in 1927, the author discussed the construction of a curve $C_{2mn}$ of order $2mn$ by the use of two pencils of lines with centers $A_1$, $A_2$, related by means of two base curves $C_n$, $C_m$ of orders $n$ and $m$, respectively, and an auxiliary pencil with center $A_3$. It was shown that the line $A_1A_3$ meets $C_m$ in $m$ $n$-fold points of $C_{2mn}$. In the present paper, it is shown that certain projective relationships yield the following method of determining the tangents at such an $n$-fold point: Project the points in which $A_1A_3$ meets $C_m$ from the point of intersection of $A_1A_3$ with the tangent to $C_m$ at the point in question, thus determining $n$ points on $A_1A_3$. These points are projected from the $n$-fold point in the desired tangents. (Received August 6, 1945.)

239. H. P. Pettit: *The tangents at certain ordinary points on a curve $C_{2mn}$.*

As shown by the author in a previous paper published in the Tôhoku Mathematical Journal in 1927, a curve of order $2mn$ is generated by means of two pencils of lines related by means of two base curves of orders $m$ and $n$ respectively and an auxiliary pencil. The generated curve was shown to pass through all common points of the base curves. This paper discusses a method of constructing the tangents to the generated curve at these common points. There is shown to be a projective relationship between the tangents to the base curves at the common point and the tangent to the generated curve, which results in the following. If $A_1$, $A_2$, $A_3$ are the vertices of the first, auxiliary, and second pencils, and $P$ is the common point of the base curves $C_1$, $C_2$, determine $K_1$ in which the tangent to $C_2$ at $P$ meets $A_2A_3$ and the point $K_2$ in which the tangent to $C_1$ at $P$ meets $A_1A_3$. Draw the line $K_1K_2$ meeting $A_1A_3$ in $T$. The line $PT$ is the required tangent to the generated curve. (Received May 26, 1945.)

LOGIC AND FOUNDATIONS


It is shown that the interpretation by the intuitionistic truth notion of realizability of Kleene (Bull. Amer. Math. Soc. abstract 48-1-85) satisfies certain formal systems of intuitionistic number theory. Further results are obtained which complete reasoning outlined by Kleene (loc. cit. and Trans. Amer. Math. Soc. vol. 53 (1943) pp. 41–73,
§16) to show that intuitionistic number theory admits, besides the extension which
gives classical number theory, also an intuitionistic extension. Both extensions are
simply consistent if the unextended system is, and the two extensions are contradic­
tory to each other. The work involves formalization of the theory of certain primitive
recursive predicates $R_k(e, x_1, \ldots, x_k, y)$ which in a formal intuitionistic system
afford a representation of the theory of general and partial recursive functions and
predicates. The present results combine with reasoning of Kleene (Abstract 48-1-85)
to establish the independence of certain formulas of the intuitionistic predicate cal­c
ulus, in particular of the formula $\forall x (A(x) \lor \forall x A(x))$. (Received August 8, 1945.)

STATISTICS AND PROBABILITY


The stochastic limits of certain functions of random samples are determined where
the samples are taken from different distributions belonging to a continuous family of
distributions. (Received August 22, 1945.)


Let $p_{ij}(t), i, j = 1, 2, \ldots, 0 \leq t < \infty$, be the transition probability functions of a
Markoff process. Let $x(t)$ be the (integral) value assumed by the probability system
at time $t$. Necessary and sufficient conditions are found that the $p_{ij}(t)$ satisfy the sys­tems
of first order differential equations (*) $p_{ik}'(t) = -q_{ik} p_{ik}(t) + \sum_{j \neq k} q_{ij} p_{jk}(t)$, $p_{ik}(0)$ is made of the processes for which the discontinuities of $x(t)$ are well ordered. It is
shown that if $q_{i0}$ are specified arbitrarily except that $\sum q_{i0} = 0$, there is always a corresponding set of functions $\{p_{ij}(t)\}$ determining a Markoff process, but
that in general there will be infinitely many such sets of functions, and even infinitely
many satisfying (*), such that the discontinuities of $x(t)$ are well ordered. The initial
conditions $p_{ij}(0) = \delta_{ij}$ are thus insufficient to determine uniquely the solutions to (*).
(Received September 22, 1945.)


Let $x(t)$ be an element of the Wiener space. It is shown that the average of the func­
tional $\mathbb{E}(z f(x(t)) dt)$ ($z > 0$) is given by the formula $\sum_{i}^{2} \frac{1}{\kappa_{i}} \exp \left( -\frac{z}{2} \kappa_{i} \right)$, where $\delta_{1}, \delta_{2}, \ldots$ are positive zeros of the derivative of $P(v) = (2v)^{2/3} J_{-1/3}(2^{2/3}v^{3/2})$ and $\kappa = (1 + \int_{0}^{\infty} P(v) dv)/\delta_{i}$. A related limit theorem in calculus of probability is discussed. In the course of the proof the following seemingly
new result was also obtained: If $r_{i}$ is the jth positive root of $J_{\nu}(x)$ ($v \geq 0$) then
$r_{j}^{2} > v^{2} + (2\pi)\nu^{2} v^{4/3}$. For $j$ fixed the estimate is weaker than a known asymptotic for­mula. The value of the estimate is due to the fact that $j$ can depend on $v$. (Received
August 2, 1945.)

244. Isaac Opatowski: Calculation of Markoff chains by incomplete
gamma and beta functions and by Charlier polynomials.

Several types of stochastic processes consisting of successive transitions between $n$
states $\{i\}^n$, are considered (cf. Bull. Amer. Math. Soc. vol. 51 (1945) p. 665). Call $P_{r,s}(t)$ the probability of being a system at the time $t$ in the state $s$ if it is at $t = 0$ in
the state $r$. Let the only transitions possible during $dt$ be $(i \rightarrow i-1) (i+1 \rightarrow i)$ and the