

be indispensable to every worker in the theory of rings, but may also be used in connection with an introductory course in abstract algebra.

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An essay on the psychology of invention in the mathematical field. By Jacques Hadamard. Princeton University Press, 1945. 143 pp. \$2.00.

Professor Hadamard points out at the beginning of his little book that he is handicapped in the study it deals with by not being a psychologist. Perhaps I should point out that I am handicapped in reviewing him by being neither a psychologist nor a mathematician. But as he bravely goes on, so must I; both of us converging on that question of extraordinary interest in the history of ideas: How do great discoveries and inventions come about?

Hadamard's answer—limited, of course, to the mathematical field—is based on a variety of evidence: the testimony of contemporary mathematicians, the writings of previous psychologists, philosophers and scientists, the interpretation of certain characteristics (logical or intuitive) in the work of famous discoverers and, finally, the author's own minute introspection.

From a careful analysis and comparison of these diverse materials, Professor Hadamard concludes that the general pattern of invention, or, as it might also be put, of original work, is three-fold: conscious study, followed by unconscious maturing, which leads in turn to the moment of insight or illumination. Thereupon another period of conscious work ensues, the purpose of which is to achieve a synthesis of several elements: the novel idea, its logically deduced consequences including proof, and the traditional knowledge to which the new item is added.

Hadamard's investigation, modest and tentative as are its results, seems to me of capital importance in the realm of criticism and cultural history. For what he has done is to show that the human mind tends to behave much the same way whenever it invents, whether in mathematical or in poetic form—a conclusion which does not deny differences of temperament. Our author, on the contrary, is at pains to distinguish among types of mathematical geniuses. He classes them as logical or intuitive, concrete or abstract, yet with enough flexibility to allow for deceptive appearances and for the overlapping of categories. But it is clear in the end that in any process of creation there lurks a mystery—a mystery at least equal to that of thinking itself.

It is worth noting that Hadamard is ever ready to accept as sidelights on his subject the reports of a Mozart or a Paul Valéry on their

respective arts. In other words, the customary distinction between mathematics, "cold," "logical," "precise," and the fine arts, presumably hot, chaotic, and gaseous, disappears on a really close view of what goes on. Historical examples show how mathematical genius has leaped ahead of funded truth and thereby become unintelligible to its contemporaries (Galois), just like the "modern" artists of any period; and again, how defiance of common sense and logic has occasionally led to a magnificent expansion of the field (Cardan) though incurring the charge of madness, just like a revolutionary artist, a Blake or a Berlioz. The recurrent adjective "beautiful," which mathematicians have been fond of using since Poincaré, is therefore not misplaced. Rather, it serves as a reminder of the real bond uniting mathematical (that is, scientific) and artistic creation.

So useful and substantial is Professor Hadamard's book, and so compact as well, that one feels a certain reluctance to isolating a few defects in its composition. Yet in the interests of further research, this must be done. A first and usually superficial but constant fault is its misuse of English words cognate with French. Since the author thanks his publishers for their help, I presume they made suggestions as to his English; but these were inadequate in kind and number, leaving the text here and there ambiguous for any reader who does not guess the original thought in the author's mind.

A possibly related error, but more serious, is the misinterpretation of what William James says in a passage quoted from *A pluralistic universe*. Oddly enough, Hadamard is blaming James's narrow conception of logic as verbalized thought when, in fact, the impugned passage precisely asserts James's independence of that conception.

Still with respect to sources and quotations, I regret that Professor Hadamard's attempt at covering "the literature" should have led him to deal with a good many second-rate French writers of the late 19th century. Souriau and Paulhan, even Ribot, Fouillée and LeDantec, are scarcely impressive, either in reputation or in actual utterance. Nor does one find much new light in the suggestions culled by the author from contemporaries such as Valéry and A. E. Housman. Again, Graham Wallas in his *Art of thinking* was only a high class popularizer; his ideas come from James and others who should be preferred as authorities. Similarly, it would have been better to quote Freud on the unconscious instead of Dr. de Saussure. This methodological error only *thickens* the essay; in the room occupied by the epigoni, one could have had the weightier words of the great artists and psychologists, matching the scientists, whom Hadamard is careful to take only from the first rank.

Both because of these minor blemishes and for the sake of wider influence, it is to be hoped that the author will return to his theme and at once revise and expand his essay. There exists, outside mathematics, a considerable literature of "creators' confessions" in which he would find support and amplification of his results.

JACQUES BARZUN

Table of arc sin x. Prepared by Mathematical Tables Project, National Bureau of Standards. New York, Columbia University Press, 1945. 19+124 pp. \$3.50.

The principal table lists the values of $\arcsin x$ to twelve decimal places at intervals of .0001 for the range $0 \leq x \leq .9890$ and at intervals of .00001 for $.9890 \leq x \leq 1$. The second central difference is also tabulated. The methods and accuracy of interpolation are discussed in the introduction. For example, using the Gregory-Newton formula through second differences one may obtain to twelve decimals the arc sine of an argument (in a suitable range) given to seven decimals. This operation may be carried out conveniently on a ten-bank calculating machine. There are six short auxiliary tables which facilitate interpolation.

E. R. LORCH

Tables of associated Legendre functions. Prepared by Mathematical Tables Project, National Bureau of Standards. New York, Columbia University Press, 1945. 46+306 pp. \$5.00.

Work on these tables was begun in 1940 to meet an urgent need for a table to six significant figures at intervals of 0.1. As here presented, the tables are subject to difficulties in interpolation in the neighborhood of certain values of the argument. The Mathematical Tables Project hopes to carry out a subtabulation program which will eliminate these difficulties. Since the date of completion of this program is very uncertain, the tables so far completed are herewith made available.

There are fourteen principal and five supplementary tables. For example, table I gives the values of $P_n^m(\cos \theta)$ for values of n from 1 to 10, of m from 1 to 4 ($m \leq n$) and of θ from 0° to 90° in intervals of 1° ; results are given to six significant figures. Also tabulated are $dP_n^m(\cos \theta)/d\theta$, $P_n^m(x)$, $1 \leq x \leq 10$; $Q_n^m(x)$, $1 < x \leq 10$; its derivative; and other variations of these functions involving pure imaginary arguments and half integral values of n . Dr. Lowan has written an