ON MERSENNE'S NUMBER M_{199} AND LUCAS'S SEQUENCES

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On July 27, 1946 the writer finished calculating the 198th remainder of the Lucasian sequence 3, 7, 47, $\cdots$ as applied to the 60-digit Mersenne number $2^{199}-1=80346$ 90221 29495 13777 09810 46170 58130 12611 01496 89139 64176 50687. The result was $r_{198}=8387$ 51186 96313 46717 54322 73509 44243 96183 21834 95333 72125 49353. Since this residual is not zero and since the calculations were performed with great care it follows that M_{199} is composite.

During the course of the work each arithmetical operation was checked with the auxiliary moduli 10^5+1 and 10^8+1. After the date given above all of the work-strips of the entire set were again examined and checked with a convenient modulus. As explained in an earlier paper[1] the essential figures of each of the terms above the 8th of the Lucasian sequence for $p=4n-1$ were multiplied in order by the reciprocal of the chief modulus, M_{199}, in preference to direct division by M_{199}. The approximation to this reciprocal was computed to be $(1/M_{199})_a=0.$(59 zeros) 12446 03055 57222 83414 28812 81075 60248 48118 05043 37442 33426 62022 48719 94705 70653 43858 15449 04227 91658 81751 47907 01374 27370 69153 20476 82353 07955 41810 78545 10066 37179 54983 56718 66249 21399 25125 81295 76504 91223 78627 47138 87. This terminated reciprocal was checked by multiplying it by M_{199}. The product $(M_{199})\times(1/M_{199})_a$ equaled $1+(3.41924 \cdots)\times 10^{-207}$ which indicates a positive error of about 0.425 of a unit in the last figure (7) of $(1/M_{199})_a$. In the present work only the first nine octads (72 significant figures) of $(1/M_{199})_a$ were required. The remaining figures of this reciprocal were computed in order to cover fully the possibility of repeating the investigation by the alternative method[2] in which all quotients are omitted.

For future work and for comparison with the results of others it may be appropriate to record in this place the values of the ninth terms of the Lucasian sequences corresponding to $p=4n-1$ and $p=4n+1$ ($M=2^p-1$, p prime) as computed by the author. For the first sequence 3, 7, 47, $\cdots$ we have $\sigma_9=100$ 38568 98919 21376 68875 42399 92826 25670 48796 27683 18190 15150 99398 61346 56188 84806 97130 40351 21947 36890 55940 88447. This term would

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[1] H. S. Uhler, *First proof that the Mersenne number M_{167} is composite*, Proc. Nat. Acad. Sci. U.S.A. vol. 30 (1944) pp. 314–316.

[2] Ibid. p. 315.

cover all $(4n-1)$-primes up to, and inclusive of, $p=347$. Strictly speaking[3] Mersenne's numbers end with $p=257$. For the second sequence 4, 14, 194, $\cdots$ we have $s_9=26$ 21634 65049 27851 45260 59369 55756 30392 13647 87755 95245 45911 90600 53495 55773 83123 69350 15956 28184 89334 26999 30798 24186 64943 27694 39016 08919 39660 72975 85154. This term would be applicable to *all*[4] odd primes inclusive of $p=479$.

There now remain just two numbers of the form 2^p-1 in the Mersenne range whose character has not been investigated. These are M_{193} and M_{227}. The writer has begun the study of M_{227} with the sequence 4, 14, 194, $\cdots$.

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[3] R. C. Archibald, *Mersenne's numbers*, Scripta Mathematica vol. 3 (1935) pp. 112–119.

[4] D. H. Lehmer, *On Lucas's test for the primality of Mersenne's numbers*, J. London Math. Soc. vol. 9–10 (1934–1935) pp. 162–165.

ON THE FACTORS OF $2^n \pm 1$

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A recent investigation concerning the converse of Fermat's theorem disclosed that the fundamental table of Kraitchik [1][1] giving the exponent of 2 modulo p for $p<3\cdot 10^5$ contains numerous errors[2] in the previously unchecked region above 10^5. Hence it was decided to make an independent examination of primes, considerably beyond 10^5, having small exponents. As a by-product of this search the following new factors of $2^n \pm 1$ ($n \leqq 500$) were discovered. This list is intended to supplement the fundamental table of Cunningham and Woodall [1]. The entries can be inserted in the blank spaces provided in that table. It is believed that all factors under 10^6 have now been found.[3] Moreover, any further factors of 2^n-1 for $n\leqq 300$ or of 2^n+1 for $n \leqq 150$ lie beyond 4538800. The methods used to obtain these results will be described elsewhere.

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[1] Numbers in brackets refer to the bibliography at the end of the paper.

[2] A partial list of these will appear shortly in Mathematical Tables and Other Aids to Computation.

[3] Including, of course, the previously published addenda to Cunningham and Woodall [1] which are to be found in Kraitchik [3] and [6].