ADDITION TO MY NOTE ON SEMI-SIMPLE RINGS

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In my Bulletin note on semi-simple rings\textsuperscript{1} I made use of the following definition of the radical of a ring which I attributed to C. Chevalley: "The radical of a ring $A$ is the intersection of the annihilators of all simple $A$-modules." Recently N. Jacobson has called my attention to the fact that the radical thus defined coincides with the one considered by him,\textsuperscript{2} and that Chevalley's statement can easily be shown to be equivalent with the following characterization of the radical by Jacobson:\textsuperscript{2,3} "If $A$ is not a radical ring, then the radical of $A$ is the intersection of all the primitive ideals contained in $A$.

To see the relation between the two statements, we need to make use of Jacobson's characterization of a primitive ideal as a proper ideal $B$ such that the factor ring $A/B$ is isomorphic with a simple ring of endomorphisms. From this it is clear that $B$ is primitive if, and only if, $B$ is proper and is the annihilator of a simple $A$-module. If the word "proper" is dropped from the definition of a primitive ideal, then the concept of primitive ideal is equivalent to that of annihilator of a simple $A$-module. Hence Chevalley's definition is essentially the same as Jacobson's characterization.\textsuperscript{4}

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\textsuperscript{1} A characterization of semi-simple rings, Bull. Amer. Math. Soc. vol. 52 (1946) p. 1021.
\textsuperscript{2} N. Jacobson, The radical and semi-simplicity for arbitrary rings, Amer. J. Math. vol. 67 (1945) p. 301.
\textsuperscript{4} It is to be noted that, as a result of this equivalence, Theorems I and II of my note become superfluous. See Theorems V, IX, and XXV in footnote 2 above.